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Research Article

PROPORTIONAL REASONING: INSIGHTS FROM STUDENTS' EARLY ATTEMPTS AT QUANTIFYING IN SOLVING MATHEMATICAL LITERACY PROBLEMS

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Abstract
This research aims to explore students' proportional reasoning at the early attempts at quantifying stage when students face proportional situations, especially mathematical literacy problems. This research uses a qualitative research approach involving grade 9 junior high school. Data was collected by providing mathematical literacy problems containing proportional situations and in-depth interviews with research subjects. The research results show that students in the early attempts at quantifying stage begin to identify important variables, recognize regularities, and develop plans to solve problems. Students apply inappropriate addition relationships in proportional situations, add or subtract numbers without understanding the concept of comparison, and have a sense of comparing sizes that are bigger and smaller, but students have limitations in understanding the concept of proportionality, especially in recognizing when comparison should be used, students try to identify comparative patterns, students have difficulty connecting patterns to multiplicative forms; able to conclude, but the conclusions are wrong due to limitations in understanding the concept of proportionality which results in inaccurate results. This study provides a novel contribution by specifically focusing on the "early attempts at quantifying" stage, a crucial yet underexamined phase in proportional reasoning development. The findings fill a gap in understanding how students transition from additive to multiplicative reasoning when faced with real-world mathematical problems. The implication is that assistance is needed in the form of exercises that increase students' understanding and sensitivity to multiplicative situations and relationships between variables in solving mathematical literacy problems.

Keywords: Early Attempts at Quantifying, Mathematical Literacy, Problems, Proportional Reasoning, Solving

INTRODUCTION

Proportional reasoning is a very important ability in involving mathematics in understanding the comparison between two quantities or more (Copur-Gencturk et al., 2023; Nelson et al., 2022; Nugraha et al., 2023). Proportional reasoning related to thinking multiplicative (Prayitno et al., 2018; Elien Vanluydt et al., 2020; Khasawneh et al., 2022; Temizer, 2022; Rachmanto, T. B., & Akande, 2024). Students start to understand ratio as marking the only one that can used for different situations However proportional, students must Study recognize relationships in different situations or Study that inside dual capacity situations have the same ratio.

Proportional reasoning involves understanding the multiplicative relationship between rational quantities, namely $\frac{a}{b} = \frac{c}{d}$, which is a common form commonly used in everyday life (Arican, 2019; Frith & Lloyd, 2016; Im & Jitendra, 2020). Students must be able to compare situations where the measurements are not in the same ratio and decide how the ratios differ because most proportions do not involve solving proportions at all but rather comparing ratios in situations that are similar but not proportional (Walle, 2007). Recognizing multiplicative relationships involves understanding the concept of multiplication and the ability to identify patterns involving multiplication operations between numbers (Amador et al., 2024; E. Vanluydt et al., 2023).

The ability to think proportionally is seen as the main indicator of the level of formal operational thinking, which is considered the highest stage of cognitive development (Randewijk, 2010). According to Fielding-Wells et al., (2014), the development of proportional reasoning is a stage that is supported by the development of increasingly complex multiplicative thinking and the skill to compare two quantities multiplicatively. Research conducted by Baxter & Junker (2001) shows that there are five stages of development of proportional reasoning, namely 1) qualitative, 2) early attempts at quantifying, 3) recognizing multiplicative relationships, 4) accommodating covariance and invariance, and 5) functional and scalar relationships. However, the research that has been conducted has limitations, namely limited testing time and difficulties in recording informative behavior from each student.

The second stage, namely the stage of early attempts at quantifying, can be interpreted as initial steps to measure quantitatively or initial efforts to quantify, covers the initial stages or initiatives in developing methods to measure, assess, or express an idea in numerical form. Students often engage in constant additive differences (for example, a - b = c - d) rather than multiplicative relationships. Students still rely on calculating increases or decreases (Baxter & Junker, 2001). The results of identification are limited, by giving questions to students, when students solve questions in the process of using mathematical concepts, facts, and procedures, proportional reasoning occurs at the early attempts at quantifying.

In the early attempts at quantifying, students' answers will likely be measured using a constant additive difference compared to a multiplicative relationship called the addition strategy (Baxter & Junker, 2001; Halimah et al., 2024; Zakiyah et al., 2024; Anugradia et al., 2025). Before understanding A strong understanding of multiplicative relationships becomes very important in solving various types of mathematical problems, including mathematical literacy problems, which involve the use of proportional reasoning to solve everyday challenges which are also included in the Program for International Student Assessment (PISA) topic. Proportional reasoning is related to mathematical literacy, there is a strong common thread between proportional reasoning and mathematical literacy, as revealed in Randewijk's (2010) research.

PISA transforms the principles of mathematical literacy into three components, namely: content, process, and context (Hendroanto et al., 2018; She et al., 2018). According to Stacey (2015), mathematical content components include space and shape; quantity; change and relationships; as well as uncertainty and data. Real-world context components include scientific; personal; work and society. Mathematics students can cultivate logical and systematic thinking, enabling them to analyze problems or situations, formulate hypotheses, plan effectively, make informed decisions, and solve problems, all of which can be applied to every day (Syutaridho et al., 2023; Yohanie et al., 2023 Kamid et al., 2024; Sarnoko et al., 2024; Firmansyah et al., 2024). Process components include formulating, implementing, and interpreting. Mathematical literacy problems according to PISA aim to measure how students apply their mathematical knowledge in solving problems in various real contexts (H Julie et al., 2019; Citrawan et al., 2024; Habibi et al., 2024; Mardiati et al., 2024; Miharja et al., 2024).

The results of the assessment carried out by PISA were not encouraging results for Indonesia, in 2018 Indonesia was ranked 72nd out of 78 countries (Kholid et al., 2022; Schleicher, 2019). Finally, in 2023 Indonesia was only ranked 69th out of 81 participants (OECD, 2023) compared to neighboring participating countries such as Malaysia, Brunei Darussalam, and Singapore. Malaysia is ranked 54th, Brunei Darussalam is ranked 40th, while Singapore is very superior at rank 1. Several previous studies have revealed the low level of mathematical literacy in Indonesia (Harsiati & Priyatni, 2017; Hongki Julie et al., 2017; Larasati & Rianasari, 2017; R. H. N. Sari & Wijaya, 2017). According to Putra & Vebrian (2020), Indonesian students still experience difficulties in answering mathematical literacy problems according to PISA questions.

The PISA questions tested include percentages, ratios, and proportions, which involve understanding the relative relationships between quantities as well as the use of proportions and proportional reasoning in solving mathematical problems (OECD, 2021). Proportional reasoning can be revealed when solving problems presented in the form of mathematical literacy problems. For example, the OECD (2022) presents the question "What is the relationship between the distance traveled, d, in kilometers, and the time spent driving, t, in hours, if you drive at a constant speed of 108 km per hour?". This emphasizes the importance of proportional reasoning in solving mathematical literacy problems, which require reasoning abilities, including proportional reasoning as one type of relevant reasoning. Mathematical literacy problems were chosen because they are questions that are made under the standards provided by PISA. In mathematical literacy problems, many questions involve proportional reasoning to solve the problems given.

Research on proportional reasoning has been extensively explored by various researchers (Ojose, 2015; de la Cruz & Garney, 2016; Misnasanti et al., 2017; Johar et al., 2018; Arican, 2019; Im & Jitendra, 2020; Thurn et al., 2022; Hernández-Solís et al., 2023; Sari et al., 2023; Y. M. Sari et al., 2023; Supply et al., 2023; Laksono et al., 2025). These studies largely focus on students' difficulties in understanding the concepts of ratio and proportion. However, as Arican (2019) points out, while these studies address the obstacles students face, they often provide limited insights into students' specific strengths and weaknesses. Despite the abundance of research on proportional reasoning, there remains a gap in understanding how students develop proportional reasoning skills, particularly when applied to mathematical literacy tasks at the "early attempts at quantifying" stage. Mathematical literacy, which emphasizes real-world problem-solving and application of mathematical concepts, provides an essential context for assessing students' proportional reasoning. The novelty of this research lies in addressing this gap, as no prior studies have specifically examined students' proportional reasoning in this early stage of quantification within the framework of mathematical literacy.

This study builds on the foundational work of Baxter & Junker (2001), OECD (2022), and Randewijk, 2010). Randewijk (2010) underscores the significant link between proportional reasoning and mathematical literacy, highlighting the importance of enhancing proportional reasoning skills within literacy-based mathematics education. OECD (2022) includes mathematical literacy problems that require proportional reasoning, especially in the early attempts at quantifying stage. Baxter & Junker (2001) identified the stages of proportional reasoning development, namely: 1) the qualitative stage, 2) early attempts at quantifying, 3) recognition of multiplicative relationships, 4) accommodating covariance and invariance, and 5) functional and scalar relationships. The second stage, "early attempts at quantifying," is particularly crucial. However, their study had limitations, such as time constraints and insufficient observation of student behavior, leaving key aspects unaddressed. These gaps point to the need for a more comprehensive analysis of students' proportional reasoning during this early stage.

The novelty of this research lies in its specific focus on the "early attempts at quantifying" stage of proportional reasoning within the context of mathematical literacy problems. While previous studies have examined proportional reasoning broadly, this study targets a previously unexamined stage of reasoning development in a real-world problem-solving context. By focusing on this unique intersection, this research seeks to provide new insights into how students engage with and apply proportional reasoning during the early attempts at quantifying stage, offering a more nuanced understanding of their cognitive processes in mathematical literacy.

RESEARCH METHOD

The research approach used in this research was qualitative. Qualitative research is research that aims to explore problems and develop a detailed and in-depth understanding of the source of the problem (Creswell, 2014). This research aims to get an overview and analyze students' proportional reasoning at the early attempts at quantifying in solving mathematical literacy problems. This research was conducted at the 9th-grade junior high school in the 2022/2023 academic year, involving the participation of 191 students. Researchers chose research subjects as students who solved problems using an addition strategy. Subject selection was focused on gaining an in-depth understanding of students' experiences in solving problems in proportional situations. The process of selecting research subjects can be outlined in the research design scheme as follows: Figure 1 presents a step-by-step visual representation of the process for selecting research subjects scheme.

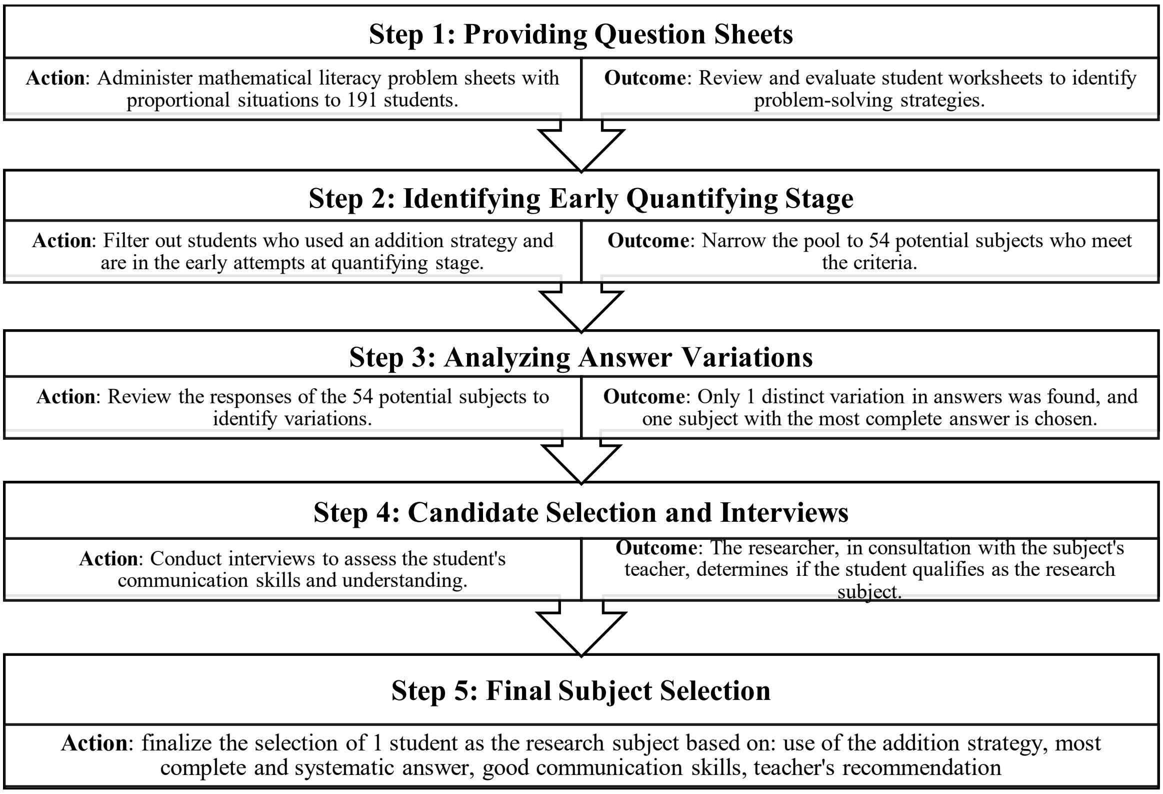


Figure 1. The process for selecting research subjects scheme

Based on the process for selecting research subjects scheme in Figure 1, specifically Step 5: Final Subject Selection, one subject was chosen to represent the 54 students who completed the early attempts at quantifying stage. This decision was made because all students provided the same answers, with only one variation. Therefore, selecting a single subject is deemed sufficient for analysis and presentation in this research article. Data was collected through a mathematical literacy test specifically designed to measure students’ proportional reasoning abilities.

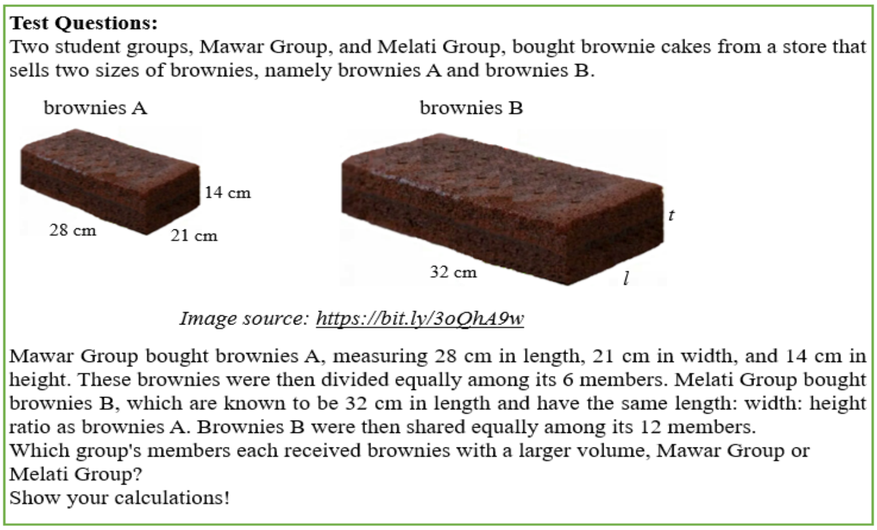


Figure 2. Mathematical literacy problems

Data was collected by providing mathematical literacy problems specifically designed to measure students' proportional reasoning abilities and in-depth interviews with research subjects. The research instruments used had to be valid (Hidayat et al., 2024); therefore, the instruments in this study were confirmed to be both valid and reliable. In-depth interviews focus on exploring and understanding the subjective experiences experienced by research subjects regarding solving problems in proportional situations (including the subject's feelings, thoughts, and perceptions). Individual semi-structured interviews were conducted with research subjects. Researchers developed an interview guide of 20 questions that students had to think about and answer to gain the necessary information. Additionally, researchers used observation sheets and documented supporting data for the study. Each interview lasted approximately 25 minutes, and all interviews were recorded and transcribed for further analysis.

Data analysis carried out in this research went through 3 stages, namely data reduction, data presentation, and conclusion. Transcription was carried out to convert recorded interviews into written text. The data was then analyzed in depth through several steps, such as identifying themes and motifs that emerged in the interviews. Researchers looked for general meanings that emerge from the experiences of research subjects and tried to understand the essence and structure of students' subjective experiences. Researchers organized and grouped themes that emerged from data analysis. Researchers sought to understand students' experiences holistically and gain an in-depth understanding of the phenomena under study. Data interpretation involves continuous reflection and analysis to develop a deeper understanding. In this study, method triangulation was also employed to compare information or data using different approaches. The researcher used interviews, observations, and surveys. To ensure the reliability and accuracy of the information, and to provide a comprehensive understanding, interviews and observations were utilized to cross-check the validity of the data.

RESULTS AND DISCUSSION

Identification of students' misconceptions

Based on the results of the mathematical literacy test specifically designed to measure students' proportional reasoning, the data were grouped according to the stages of proportional reasoning development. The test results are presented in the following Table 1.

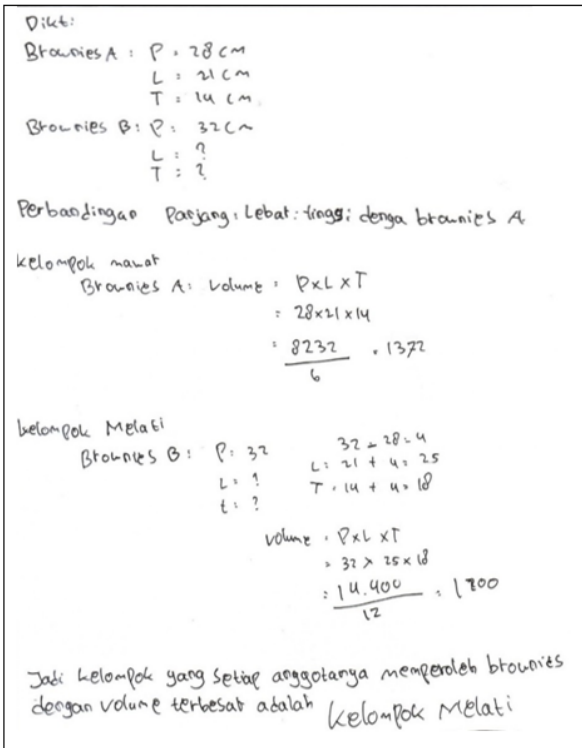
Table 1. Distribution of students from the mathematical literacy test results

No	Development Stage	Number of Students
1.	Qualitative	24 Students
2.	Early attempts at quantifying	54 Students
3.	Recognition of multiplicative relationships	54 Students
4.	Accommodating covariance and invariance	57 Students
5.	Functional and scalar relationships	2 Students

This study focuses exclusively on students' proportional reasoning at the early attempts at quantifying stage, in line with the research objectives. By concentrating on this critical stage, the research aims to provide new insights into how students engage with and apply proportional reasoning during this phase, offering a more nuanced understanding of their cognitive processes in mathematical literacy. This stage is crucial for developing proportional reasoning in subsequent stages.

Among the 54 potential subjects at this stage, only one internal variation was identified, so a single subject was selected for detailed analysis. This decision was made because all students provided similar answers, with only one variation. Therefore, selecting one subject was considered sufficient for analysis and presentation in this research article.

The working results of Subject 1 (S1) in the finish question literacy mathematical stage formulated problem showed with excerpt answer written as presented in Figure 3.

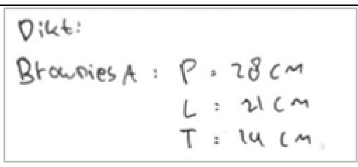


Translation:
Is known:
Brownies A: $p = 28$ cm
$l = 21$ cm
$t = 14$ cm
Brownies B: $p = 32$ cm
$l =?$
$t =?$
Comparison *length* : *width* : *height*, with brownie A
Rose Group,
Brownies A:
Volume $= p \times l \times t$
$= 28 \times 21 \times 14$
$= \frac{8232}{6} = 1372$
Jasmine Group,
Brownie B
$p = 32$ cm $\quad 32 - 28 = 4$
$l =?$ $\quad l = 21 + 4 = 25$
$t =?$ $\quad t = 14 + 4 = 18$

Volume $= p \times l \times t$
$= 35 \times 25 \times 18$
$= \frac{14.400}{12} = 1{,}200$
So, every group its members get the largest volume of brownies is Jasmine Group

Figure 3. Results of undergraduate work in solving mathematical literacy problems

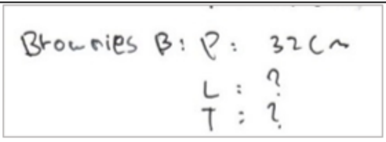
At the stage of formulating a situation mathematically, S1 began with students identifying important variables; and recognizing mathematical structures, namely order, relationships, and brownie patterns. A. The results of S1's work in solving mathematical literacy problems at the problem formulation stage are shown in excerpts from written answers presented in Figure 4.



Translation:
Is known:
Brownies A: $p = 28$ cm
$l = 21$ cm
$t = 14$ cm

Figure 4. S1's answers at the Identifying Brownies A stage

S1 started by identifying what was known about brownie A, then wrote down the known dimensions of brownie A, namely *length* (p) was 28 cm, *width* (l) was 21 cm, and *height* (t) was 14 cm. The results of the student's work showed that S1 had recognized mathematical structures, regarding regularity, relationships, and patterns of size arrangement of brownies A. Next, S1 analyzed brownies B, while the identification results for brownies B are presented in Figure 5.



Translation:
Brownies B: $p = 32$ cm
$l = ?$
$t = ?$

Figure 5. S1's answer is p stage identifying brownies B

S1 wrote *length* = 32 cm, *width* = ?, and *high* = ?, this showed S1 was capable of identifying variables in brownie B and planning to look for the *width* and *height* of brownie B. In addition, S1 was capable of identifying comparison with write pattern comparison *length, width,* and *height* which has comparison the same between brownies A, as for answer written student presented in Figure 6.

Translation:
Comparison *length* : *width* : *height,* with brownies A

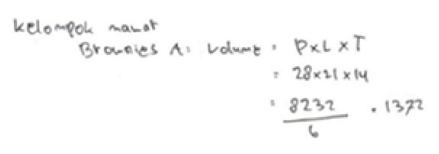
Figure 6. S1's answer is p stage write comparison patterns *length*: *width* : *height*

Following quote interview between researchers (P) and S1 carried out:

P : *Do you think brownie A is longer? Big from length from brownie B?*
S1 : *Yes, uh no*
P : *Why?*
S1 : *Brownie B is longer than the length of Brownie A, the length of brownie B is 32 cm, meanwhile, brownie A is 28 cm in length*
P : *OK, how much difference in length of brownies A* and *B*
S1 : *4 cm*
P : *So yes, that's 4 cm There is a relationship with looking for width* and *height of brownie B.*
S1 : *There isn't any*
P : *Is the width* and *height of Brownie B already known?*
S1 : *Yes, Oh not yet Not Brownie B yet*
P : *What are you planning to know about the width* and *height of brownie B?*
S1 : *Reduce brownie length B with the length of brownie A then add the result with the width* and *height of brownie A*

Based on the results analysis answers as well as interviews already done in stages formulate the situation in a way mathematics, S1 capable of 1) identifying variables important things to know about the question; 2) writing down what is known with complete; 3) recognizing regularity and arrange a plan for finish problem. 4) writing down what is necessary to search for moreover formerly; 5) making a series of regular instructions for solving problems.

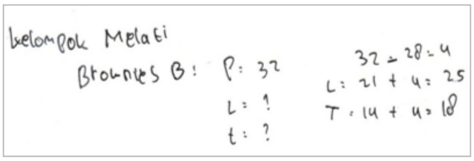
At this stage when applying procedures, facts, concepts, and reasoning mathematics, S1 found the volume of brownie A and wrote it down moreover formerly formula for the volume of a block, was known as a brownie-shaped beam. The formula used was $V = p \times l \times t$. Then S1 continued with substitute size *length* (p), *width* (l), and *height* (t) of brownies A calculation process is presented in Figure 7.



Translation:
Group:
Brownies A: Volume = p × $l \times t$
= 28 × 21 × 14
= $\frac{8232}{6}$ = 1372

Figure 7. S1's answer at the stage of calculating the volume of brownies A

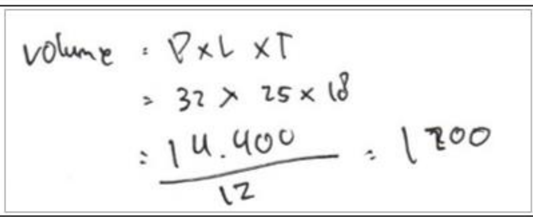
Substituting the size of the brownie into the formula for the volume of the block, we got the volume of the brownie A = 28 × 21 × 14 = 8.232. Next, S1 wrote that the volume was distributed among 6 people, so it was written $\frac{8.232}{6} = 1.372\ cm^3$. So each member of the rose group gets a brownie with a volume of $1.372\ cm^3$. Next, S1 analyzed the jasmine or brownie B group. The analysis stage carried out was to find the *width* and *height*. Determining the *width* and *height* by connecting and looking at the difference in *length* between brownies A and B, the S1 activity process is presented in Figure 8.



Translation:
Jasmine Group
Brownies B
p = 32 cm 32 - 28 = 4
l =? l = 21 + 4 = 25
t =? t = 14 + 4 = 18

Figure 8. S1's answer to the stage of determining the *width* and *height* of Brownies B

S1 connected the difference in *length* between brownies A and B with an addition relationship, it was known that the *length* of brownie A was 28 and the *length* of brownie B was 32, the difference was 32 – 28 = 4, this 4 cm difference was used to find the *width* and *height* of brownie B. S1 found *width* by writing *width* = 21 + 4 = 25, and *height* = 14 + 4 =18 so that brownie B was 32 cm *length*, 25 cm *width* and 18 cm *height*. S1 used an addition relationship rather than a multiplication relationship, so the *width* and *height* measurements sought were incorrect. After knowing the *length*, *width,* and *height* of brownie B, S1 looked for the volume of brownie B, the process of calculating the volume of brownie B is presented in Figure 9.



Translation:
Jasmine Group
Brownies B
p = 32 cm 32 - 28 = 4
l =? l = 21 + 4 = 25
t =? t = 14 + 4 = 18

Figure 9. S1's answer at the stage of calculating the volume of brownies B

S1 found the volume of brownie B with the write method, moreover formerly formula for the volume of a block, ie $V = p \times l \times t$. Next S1 started by substituting the size *length, width,* and *height* of brownie B searching for previously, obtained $V = 32 \times 25 \times 18 = 12.288\ cm^3$. After that, S1 divided the volume of Brownie B into every member group of 12 people, so written $\frac{14.400}{12} = 1.200\ cm^3$. As a result of the error in finding the volume of brownies B, the conclusion of each member of the jasmine group was wrong, the correct answer should be $1.074\ cm^3$.

P : *OK, indeed. How long is Brownie B?*
S1 : *Brownie B is 32 cm in length.*
P : *How long is brownie A?*
S1 : *The length of the brownie A 28 cm length*
P : *So how much? earlier is the difference?*
S1 : *4 cm*
P : *Continue to use for what that?*
S1 : *Used to know the width* and *height of Brownie B*
P : *So basically you conclude that 32 - 28 results in 4, then later the width is added, the result is added to get the width l, then this width is added to find the height, In your opinion, there is a connection between brownie A and brownie B.*
S1 : *There isn't any relationship, because of each group.*
P : *Okay the lengths of brownies A* and *B are 28* and *32 respectively You see There is the*

same pattern width and *height of brownies A* and *B? If linked with a difference of 4 cm in addition What is multiplication?*

S1 : *Addition, yes*

P : *Do you think the width of brownie A, which measures 21 cm, is related to brownie B in the same way as his height?*

S1 : *Yes*

P : *In touch, earlier relationship What yeah?*

S1 : *Addition of a difference of 4 cm*

P : *What is your strategy? Use the can finish existing problems?*

S1 : *Yes*

P : *Have you already carried out the calculation process?*

S1 : *Already*

P : *Explain how to calculate it. OK, if want to Look here What are you doing moreover formerly for those who write here determine the length, width,* and *height of brownie B like that right You write*

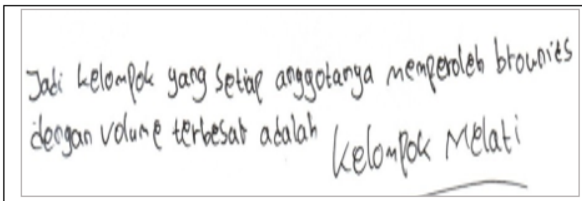
S1 : *No*

P : *Written yeah. Determine width* and *height How?*

S1 : *The method is from 28 to 32, right? The difference is still 4 I use those 4 so 21 plus 4 later the result is 25, that is I make it wide from brownies. Later another place was added 14 the result was 18, I made it as tall as the Mean brownie's width 21 + 4 = 25* and *14 + 4 = 18 division of brownies A* and *B Can be obtained so the formula plt means the volume of brownie A is 28 × 21 × 14 results Finally is at 8,232 cm³, for calculates 35×25×18 results finally 14,400 cm³ for distribution of brownies A* and *b to each member calculated from the volume shared with members, the volume of brownie A is 8,232 cm³ divided by 6 members the result is 1,372 cm³ per member whereas, for brownies B 14,400 divided by 12 results finally 12,200 cm ³ per member, So member group rose to get more brownie pieces Lots than group jasmine.*

Based on the analysis of answers and interviews conducted at the proportional reasoning at early attempts at quantifying stage, when the student was applying mathematical procedures, facts, concepts, and reasoning, it was found that S1 was able to: 1) design and choose strategies to solve problems; 2) apply the addition relationship strategy to solve the problem rather than the multiplication relationship, so that the *width* and *height* measurements sought are not correct; 3) apply number patterns to addition and subtraction operations; 4) able to perform multiplication and division operations.

Next, at the interpreting stage, the results of calculating the volume portion for each member of the rose group are compared with the volume portion for each member of the jasmine group. The conclusions drawn by S1 are presented in Figure 10.



Translation:
So, every group its members get the largest volume of brownies is Jasmine Group

Figure 10. S1's answers at the conclusion drawing stage

Based on the answers written on the answer sheet, it was found that each member who obtained a larger volume was the Jasmine group. S1's proportional reasoning in solving mathematical literacy problems as a whole to see the stages carried out by S1 to identify the emergence of proportional reasoning is presented in Figure 11.

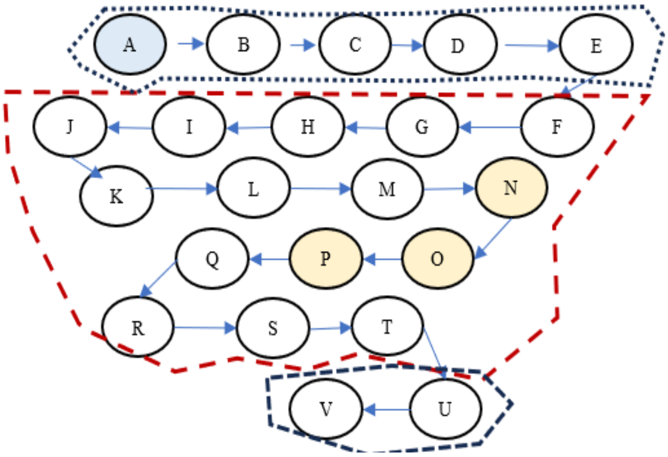
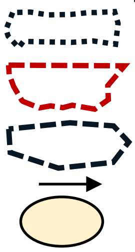


Figure 11. S1's proportional reasoning diagram in solving mathematical literacy problems

Caption Figure 10.

A: Identifying Brownies A
B: Writing down the *length* of brownie A
C: Writing down the *width* of brownie A
D: Writing down the *height* measurement of brownie A
E: Understanding mathematical structures
F: Identifying Brownies B
G: Writing *length* = 32, *width* = ?, and *height* = ?
H: Writing pattern comparison *length : width : height*
I: Looking for Volume Brownies A
J: The volume of Brownie A is 28 × 21 × 14 = 8,232 cm ^{3}
K: Looking for many parts for every member group rose
L: So every member group rose to obtain 8.232 : 6 = 1.372 cm ^{3}
M: Planning look for the *width* and *height* of Brownie B
N: Connecting the difference *length* between brownies A and B with connection summation
O: *the length of* brownie A, namely 28, and *the length* of brownie B 32, is obtained difference 32 - 28 = 4, a difference of 4 cm is used to look for the *width* and *height* of brownie B
P: Looking for *wide,* i.e. 21 + 4 = 25, and *height* = 14 + 4 = 18, so obtained size 32 cm *length,* 25 cm *wide,* and 18 cm *high*
Q: Looking for Volume Brownies B
R: Volume of Brownies B = 32 × 24 × 16= 14,400 cm ^{3}
S: Looking for many parts for every member group Jasmine
T: 14,400 cm ^{3} : 12 = 1,200 m ^{3} For every member group jasmine
U: Comparing every member group roses and every member group jasmine, which one is each member his group gets brownies with more volume big
V: Taking conclusion



: Formulating the situation mathematically

: Using concepts, facts, and mathematical procedures

: Interpret, apply, and evaluate the results of mathematical processes

: Directions for problem-solving steps

: implement relationship summation to finish the problem

Based on the analysis of students' answers and interviews, their proportional reasoning at the early attempts at quantifying stage was examined during the process of formulating the situation mathematically, S1 can 1) identify the important variables known in the problem; 2) write down what is known completely; 3) recognize regularities and develop a plan to solve the problem. 4) write down what you need to look for first; 5) create an orderly series of instructions to solve the problem. Based on the results of the analysis of answers and interviews conducted at the using stage, it was found that S1 was able to 1) design and choose strategies to solve problems; 2) apply the addition relationship strategy to solve the problem rather than the multiplication relationship, so that the *width* and *height* measurements sought are not correct; 3) apply number patterns to addition and subtraction operations; 4) able to perform multiplication and division operations. At the stage of interpreting, applying, and evaluating the results of a mathematical process, S1 concluded the results of a mathematical process; compare which size is bigger and smaller.

Based on the results of research that has been carried out, at the early attempts at quantifying stage, students did not have sensitivity to multiplicative relationships and limited knowledge about additive relationships. Students tend to use additive reasoning which is called additive reasoning disorder in proportional situations. Students should understand that proportionality only applies to ratio situations. This understanding is very important for articulating proportional relationships (Ramful & Narod, 2014). Proportional reasoning is closely related to ratio situations (Harris et al., 2015), and can be an important basis in correcting additive reasoning disorders that are still visible in the early stages of measurement.

Student performance in the early stages of proportional reasoning is usually set in the context of comparing two parts of a whole (Harris et al., 2015; Castro, 2025; Galdonez, 2025; Qiu et al., 2025; Tep et al., 2025). Proportional reasoning can be considered a psychological structure rather than a mathematical concept. The essence of proportional reasoning lies in explaining, predicting, or evaluating the relationship between two relations (or second-order relationships), not simply the relationship between two concrete objects or two quantities that can be directly perceived (Baxter & Junker, 2001). Therefore, in the early attempts at quantifying students measure using a constant difference of addition compared to a multiplication relationship which is called the addition strategy.

Although efforts to measure and perform calculations involve systematic procedures in formulating, employing, and interpreting mathematical problems, students are limited in seeing a problem that involves the addition relationship between the quantities of two objects. When trying to conclude from the results of a mathematical process with confidence, the conclusions drawn are often based on the wrong process. As stated by (Izzatin et al., 2021) many students experience difficulty in solving questions related to ratios and comparisons, and they are often confused about when to use addition or multiplication in solving them. So that students can integrate the concept of proportionality into the context of mathematical literacy so that students can see the connection between mathematics and everyday life.

In research that has been conducted Baxter & Junker (2001) at the early attempts at quantifying stage, it is explained that students often involve constant additive differences (such as $a - b = c - d$) rather than multiplicative relationships, with students only relying on calculating increases or decreases. The research results in this article provide a deeper picture of the steps, strategies, and stages used in solving problems, especially mathematical literacy problems. As for the description of the student's completion process at the early attempts at quantifying, only one internal variation was found. Based on the results of the analysis of answers and interviews conducted, it was found that students 1) identified the important variables known in the questions; 2) write down what is known completely; 3) recognize regularities and develop plans to resolve problems; 4) designing and selecting strategies to solve problems; 5) applying the addition relationship strategy to solve the problem rather than the multiplication relationship, so that the results sought are not correct; 6) apply number patterns to addition and subtraction operations. At the interpreting stage, namely 7) concluding the results of a mathematical process with confidence, but the conclusions drawn from the process are wrong. 8) can compare which size is larger and smaller, but the comparison is made from an additive relationship.

The tendency of students to apply comparisons with additive relationships in proportional situations occurs because students do not yet understand the relationship between these numbers well. In a proportional situation, the relationship between the sum or difference of the numbers must be under the existing proportion. For example, when students try to apply comparisons with additive relationships in proportional situations, students must ensure that each step of the operation corresponds to the proportions in the situation. For example, if someone is solving a proportion problem such as $\frac{2}{3} = \frac{4}{x}$, students must ensure that the addition operation they perform on the number 4 or × maintains the correct proportion, namely $\frac{2}{3}$. In addition, a tendency that can occur is to carry out mathematical operations that are inconsistent with proportions, for example, adding numbers that do not comply with the proportions or performing subtractions that do not maintain the correct proportional relationship. Therefore, it is important to understand the concept of proportion and the relationship between numbers in proportional situations and to use number patterns carefully according to the context.

In comparison with Arican (2019), who emphasized students' difficulties in distinguishing proportional from non-proportional relationships, this research provides deeper insights into how these misconceptions manifest in problem-solving processes. Additionally, the findings resonate with E. Vanluydt et al., (2023), who suggested that early proportional reasoning often involves trial-and-error strategies, reflecting students' limited understanding of proportional concepts. The generalization from this study indicates that students at the early quantifying stage require targeted interventions to bridge their understanding of additive and multiplicative reasoning. The implications are substantial for mathematics education, emphasizing the need for curriculum designs that incorporate exercises aimed at enhancing students' sensitivity to proportionality in real-world contexts (Ramful & Narod, 2014; Pillai et al., 2017; Mabeza, 2025; Sulistyani, & Mochama, 2025; Sari, & Oransa, 2025).

The novelty of this research lies in its focus on the intersection of early proportional reasoning and mathematical literacy, a relatively underexplored area in previous studies. By situating these findings in the context of mathematical literacy, this study offers actionable insights for educators to address students' reasoning gaps. This study is limited to ninth-grade junior high school students in a specific educational context, which may affect the generalizability of the findings. Future research should expand to diverse age groups and cultural settings to validate and extend these results. Additionally, incorporating longitudinal studies could provide deeper insights into the progression from additive to multiplicative reasoning over time.

CONCLUSION

Based on the research questions, findings, and discussion, it was concluded that proportional reasoning at the early attempts at quantifying stage shows that students begin to identify key variables, recognize patterns, and plan solutions to problems. However, they often apply inappropriate additive reasoning in proportional contexts, adding or subtracting numbers without fully understanding the concept of comparison. While students can compare sizes (e.g., identifying which is larger or smaller), they face challenges in understanding proportionality, particularly in recognizing when comparison should involve multiplicative reasoning rather than addition. Students also struggle to apply problem-solving strategies effectively and have difficulty connecting comparative patterns to multiplicative relationships. Although students attempt to conclude, their limited understanding of proportionality often leads to incorrect outcomes. The implication of this study underscores the need for targeted educational interventions to address this challenge. Teachers and curriculum designers should focus on integrating tasks that contrast additive and multiplicative reasoning, enhancing students' sensitivity to proportional contexts through real-world applications. By situating these findings within mathematical literacy, this research not only offers actionable insights for improving teaching strategies but also emphasizes the critical role of proportional reasoning as a foundational skill in mathematics education.

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AUTHOR CONTRIBUTIONS

Nego Linuhung: Conceptualization, instrument design, data, data analysis, and article writing. Purwanto: editing, drafting manuscript, and final approval. Sudirman: data interpretation and proofreading. Sukoriyanto: data interpretation and material support. Mohd Isha Awang: editing the manuscript and material support.

CONFLICTS OF INTEREST

The author(s) declare no conflict of interest.

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