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Research Article

(RE)CONCEPTUALIZING LINEAR EQUATIONS: A SNAPSHOT FROM TEACHING AND LEARNING IN INDONESIA

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Abstract

This study aimed to describe the zone of concept image differences in linear equations in one variable and analyze its potential impact on mathematics learning. This research was qualitative and followed a phenomenological approach. Mathematical praxeology was used to analyze the content of the knowledge to be taught (Indonesian curriculum). In contrast, didactic praxeology was used to analyze the teaching methods involved in the taught knowledge (teacher). This study explores information obtained from human and non-human sources. The object of the study was a seventh-grade mathematics textbook used in Indonesian middle schools, which refers to the *Merdeka* curriculum. The results showed differences in concept image between scholarly knowledge, knowledge to be taught, and knowledge regarding the topic of linear equations in one variable. Teachers failed to understand the information provided in the mathematics textbook that the equal sign in the concept of a linear equation in one variable represents a quantitative equation, meaning the expression on the left side of the equal sign is equal to the expression on the right side of the equal sign. This research presents an alternative praxeological reference model as an implication for the field of education in Indonesia, allowing students to generate new knowledge as justified true belief independently. Policymakers can also utilize the model to design linear equations in one variable materials that are more aligned with students' abilities by providing a structured approach that takes into account the students' prior knowledge and learning pace.

Keywords: Anthropological Theory of the Didactic, Concept Image, Mathematics Education, Praxeology.

INTRODUCTION

In mathematics, a concept image refers to the mental representation or understanding that a student has of a mathematical concept, which is shaped by their prior knowledge, experiences, and interactions with the subject (Ojo & Olanipekun, 2023; Suryadi, 2019). Research indicates that issues related to concept images in mathematics education are not confined to a single country, but are observed worldwide, varying in both how they manifest and how they are addressed. In India, students face cognitive obstacles in understanding calculus concepts, which are linked to their concept images and definitions (Alam, 2020). Future teachers in Thailand, Malaysia and the Philippines are more likely to hold this belief compared to their counterparts in the USA, Chile, Norway, Germany, and Switzerland (Özgeldi & Aydın, 2021). These beliefs can influence how teachers address concept image issues in their classrooms. This misalignment between concept images, both in students and in instructional practices, contributes directly to the learning obstacles seen in algebra learning and has led to a decline in students' mathematics performance (Fardian et al., 2024), as evidenced by international assessments like the *Programme for International Student Assessment* (PISA). The PISA results have consistently shown that students worldwide are struggling with algebra and problem-solving, which has directly impacted their overall mathematics scores (see figure 1).

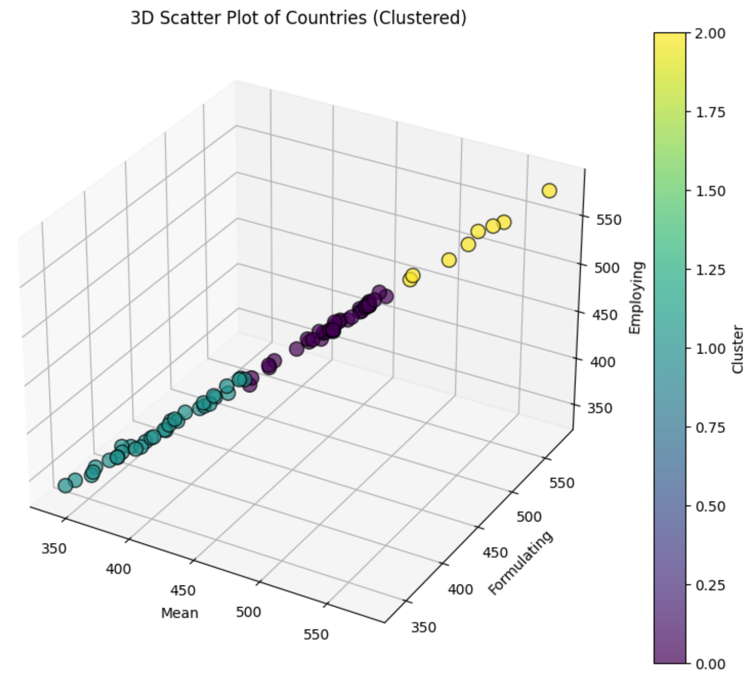

Figure 1. Global Mathematics Scores in PISA 2022 (Fardian & Dasari, 2023)

Based on Figure 1, it is identified that approximately 61% of OECD (Organisation for Economic Co-operation and Development) member countries have achieved below-average results in mathematics (represented by purple and green nodes). Overall, the PISA 2022 results indicate that nearly one-third of students in OECD countries have not reached the basic competency level in mathematics. The differences in competency distribution across each level in PISA 2022 highlight the gap in mathematical understanding, both on a national and international scale (Putri et al., 2024). This gap serves as one of the foundations for confirming that the low achievement of students in mathematics is partly due to differing concept images in understanding algebra.

Previous research on concept image has shown that the mental representations students form of mathematical symbols significantly influence their understanding and problem-solving abilities. Figure 2 visualize the fishbone diagram related to concept image in mathematics education.

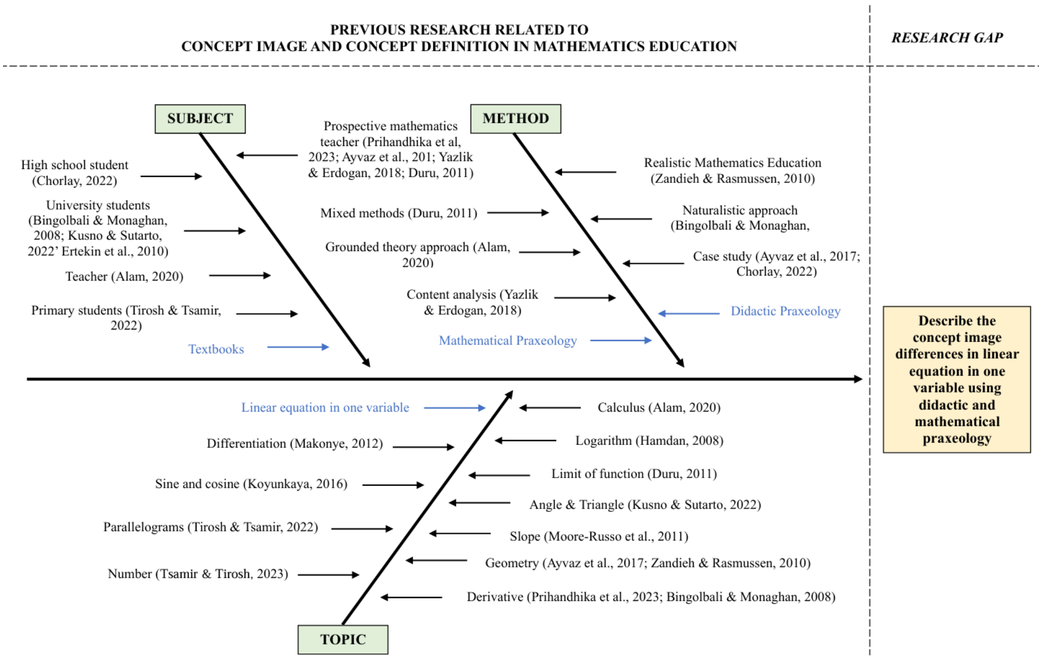

Figure 2. Fishbone diagram

Figure 2 represents previous research related to the concept image and concept definition in mathematics education, structured around three main dimensions: Subject, method and topic. Each dimension outlines various research studies conducted within mathematics education, highlighting both extensively explored areas and research gaps. Although a substantial amount of research has explored concept images in mathematics education broadly, very few studies have specifically focused on analyzing concept images in linear equations in one variable using didactic and mathematical praxeology. This unexplored area represents the novelty of the current study, offering a distinct contribution to the field by addressing how students, teachers or textbooks conceptualize linear equations in one variable within the praxeological framework.

Blanton et al. (2018) categorize early algebra into three primary areas: (1) Generalized arithmetic, (2) Equivalences, expressions, equations, and inequalities, and (3) Functional thinking. Over the past ten years, there has been a notable rise in the number of publications on early algebra research (Fardian et al., 2024). Investigations into these areas of early algebra are still relatively recent within mathematics education research. Figure 3 demonstrates that over the past five years, there has been a notable increase in research on early algebra.

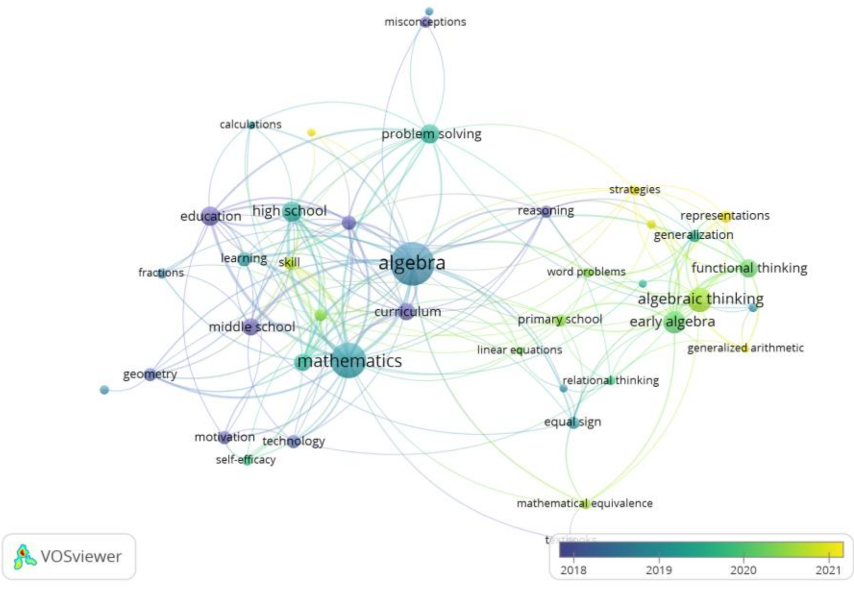

Figure 3. Overlay visualization of research trends on early algebra

Over the past five years, linear equations in one variable, functional thinking, and generalized arithmetic have remained the primary focus of research in early algebra. However, further research on linear equations in one variable shows greater potential compared to functional thinking and generalized arithmetic. Based on overlay visualization, the symbol of linear equations in one variable has the smallest size and is located farthest from the symbol of early algebra. The overlay visualization indicates a research gap, highlighting the need for further investigation into the topic of linear equations in one variable.

This study seeks to address this gap by focusing on the zone of concept image differences between scholarly knowledge, the knowledge to be taught by textbooks, and the taught knowledge by teachers. While several research has focused on individual aspects of concept image, such as student misconceptions or the role of textbooks, few studies have examined the interaction between these different sources of knowledge and how they collectively impact teaching and learning. This research aims to explore and analyze the zone of concept image differences in the context of linear equations, with implications for curriculum design and teacher training in Indonesia. By analyzing the differences in concept images across various educational stakeholders, this study provides new insights into how educational materials and teaching practices can be better aligned with students' mental representations, ultimately improving the quality of mathematics instruction in Indonesia. Additionally, the praxeological reference models proposed in this study can serve as an alternative framework for curriculum revision, ensuring that the learning process of linear equations in one variable is both systemic and epistemic, fostering a deeper understanding of the subject and its real-world applications.

Theoretical Framework

Anthropological Theory of the Didactic (ATD)

The anthropological theory of the didactic is a term introduced by Yves Chevallard to refer to a research framework in the field of mathematics didactics, which began developing in the 1980s with initial research on didactic transposition (Chevallard, 1991). ATD aims to observe human mathematical activities through an epistemological model of mathematical knowledge, primarily focusing on the didactic process of scientific knowledge from one institution to another. Figure 4 outlines the process of knowledge transposition from one institution to another (Chevallard, 2019).

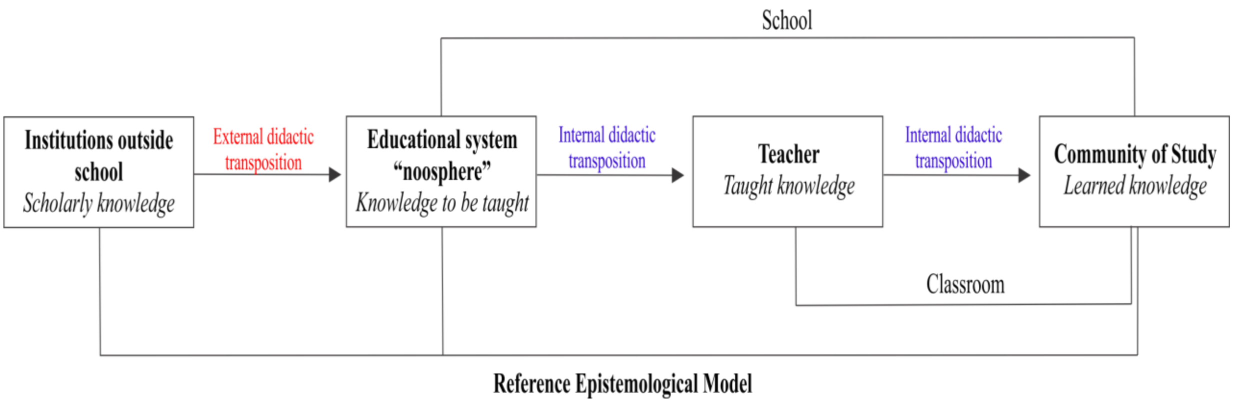

Figure 4. The didactic transposition process

Chevallard (1992) distinguishes between two types of transpositions. External didactic transposition is the shift from scholarly knowledge to knowledge to be taught. In contrast, internal didactic transposition refers to the shift from knowledge to be taught to knowledge that is truly ready to be taught (Chevallard & Bosch, 2020; Cujba, 2015; Do, 2020). However, external and internal didactic transpositions can both be analyzed using praxeology (Gök & Erdoğan, 2023; Pansell & Bjorklund Boistrup, 2018). Praxeology is the basic science that studies human actions and behaviours (Rothbard, 1997). It has developed as a central idea in ATD, the basic unit to broadly analyze actions and human behaviour. Figure 5 illustrates the four elements of praxeology.

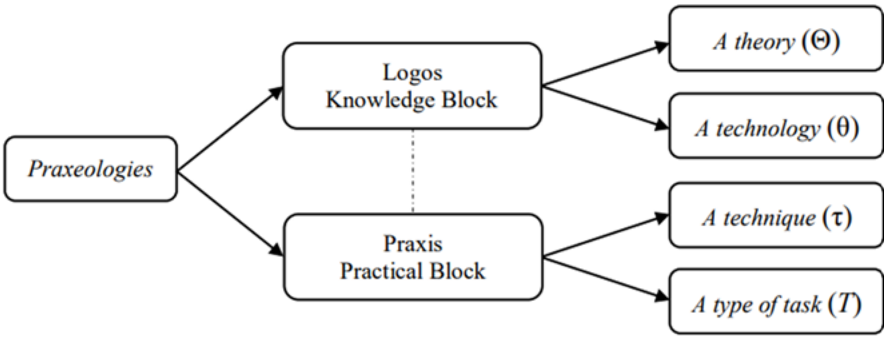

Figure 5. The elements of praxeology

Praxeology consists of two terms: *praxis*, which refers to something practical, and *logos*, which means the knowledge underlying that practicality (Huang et al., 2021). The praxis block consists of the type of task (T), which is the problem or situation given, and the technique (τ), which is the method for solving the problem. Takeuchi and Shinno (2020) divide techniques into four types. The first technique is perceptual (τ_1), which relies on visual perception or intuition to understand mathematical concepts. The second is physical (τ_2), related to using physical aids or practical demonstrations to understand mathematics. The third is operational techniques (τ_3), which focus on specific mathematical operations such as addition, subtraction or multiplication, and the fourth technique is algebraic (τ_4), which emphasizes techniques using formulas and algebraic operations for problem-solving. *Praxis* always requires logos, which includes technology (θ) to justify the techniques used and theory (Θ) to provide the foundation for that technology. Mathematical praxeology is used to analyze the content of the knowledge to be taught (Indonesian curriculum), while didactic praxeology is used to analyze the knowledge within the taught knowledge (teacher). Praxeology, as a framework for analyzing human actions and behavior in educational settings, will be crucial in understanding how concept image differences arise in the context of teaching linear equations in one variable (Fardian et al., 2025). Specifically, we will use praxeology to examine how teachers' actions, the tasks they assign and the techniques they employ influence students' concept images of linear equations in one variable.

Concept image and concept definition

In concept image and concept definition, conception plays a crucial role in shaping both aspects (Thompson & Carlson, 2017). Concept image can be understood as the entire cognitive structure associated with a concept, including mental images, properties and related processes (Tall & Vinner, 1981). Figure 2 illustrates the relationship between concept definition and other elements of the concept image and how concept definition plays a role in influencing a students' overall understanding of the concept and helps visualize the interaction between formal definition, personal interpretation and the overall concept image.

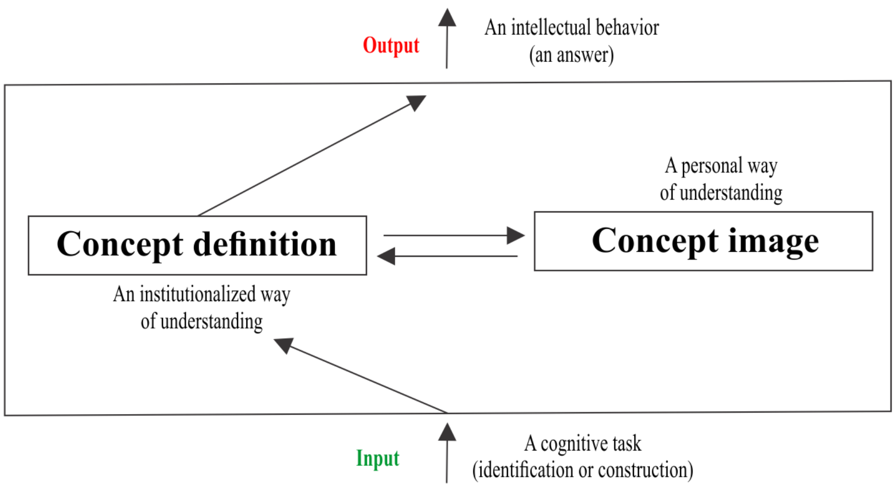

Figure 2. The relationship between concept image and concept definition (Tall & Vinner, 1981)

This misalignment between concept images, both in students and in instructional practices, contributes directly to the learning obstacles seen in algebra learning and has led to a decline in students' mathematics performance (Fardian et al., 2024; Moreno, 2022; Siagian et al., 2024). For instance, when solving linear equations in one variable, students may shift terms around using added 'rules' to obtain the correct answer without grasping the concept of balance in an equation. This procedural approach can lead to errors when they encounter more complex equations such as quadratic equations, where their methods fail.

Concept definition differs from concept image. Concept definition can be defined as a series of words used explicitly for the concept. Concept definition is a formal way to define a concept, providing clear boundaries about what is included and not included in the concept (Tsamir & Tirosh, 2023). Tall and Vinner’s (1981) framework, while useful, does not fully address the distinction between instrumental understanding (knowing-how) and relational understanding (knowing-why), which is critical for deeper conceptual understanding. In the context of mathematics, students may learn the steps to solve equations (instrumental understanding) but lack an understanding of why these steps work (relational understanding) (Putri & Juandi, 2025). This gap limits the framework’s effectiveness in fostering comprehensive conceptual learning. Therefore, to address this limitation, praxeology is integrated into this study to analyze both the *praxis* (tasks and techniques) and *logos* (technology and theory) blocks, helping to bridge the gap between procedural knowledge and conceptual understanding, and providing a more holistic view of students' learning in algebra.

Scholarly knowledge: Linear equations in one variable

An equation is an open sentence that contains the equal sign (Cohen, 1967). An open sentence is a statement whose truth value is unknown because elements are still unknown (Ajdukiewicz, 1967). The unknown elements represent all numbers and are called variables (Wagner, 1983). The numbers accompanying the variables are called coefficients, while those that do not contain variables and represent a specific number are called constants. Meanwhile, a linear equation in one variable is an equation that has only one variable, with the highest power being one (Feige & Reichman, 2004).

A linear equation in one variable is an equation on the form:

$$ax + b = 0$$

Where:

a : Coefficient ($a \epsilon R, a \neq 0$)
b : Constants ($b \epsilon R$)
x : Variable ($x \epsilon R$)

Linear equations in one variable are crucial in early algebra, as they introduce key concepts like equivalence, balance and variable manipulation, which are essential for understanding more advanced algebra. Misconceptions, such as viewing the equal sign merely as a prompt to "find the answer", can hinder students' understanding. This procedural approach, without grasping the underlying concept of balance, can lead to difficulties when solving more complex equations, creating long-term barriers to their mathematical development.

RESEARCH METHOD

This study aims to describe the zone of concept image differences in linear equations in one variable and analyze its potential impact on mathematics learning. This research is qualitative and follows a phenomenological approach. Mathematical praxeology is used to analyze the content of the knowledge to be taught (Indonesian curriculum), while didactic praxeology is used to analyze the teaching methods involved in the taught knowledge (teacher).

The research follows a structured, multi-phase approach. The first phase involves analyzing scholarly knowledge through a document study, where mathematics textbooks and related literature are reviewed. The second phase of the study involves classroom observations, aiming to analyze how teachers teach the concept of linear equations in one variable. Finally, the third phase includes diagnostic tests and in-depth interviews. These assessments are designed to validate the praxeological analyses conducted earlier, while the interviews provide deeper insights into the teacher's perspective on the concept of linear equations in one variable. The final step of the research is to integrate the findings from scholarly knowledge, mathematical praxeology, and didactic praxeology into a praxeological reference model, which serves as a framework for understanding the concept image of linear equations in one variable. To provide clarity and structure, the research process is summarized in Table 1.

Table 1. Research timeline

Research phase	Activities	Timeframe
Phase 1: Document analysis	Revieweing the textbooks and literature on linear equations	Januari – March 2024
Phase 2: Classroom observations	Observing teaching practices and techniques used by teachers	May – July 2024
Phase 3: Interview and diagnostic	Conducting in depth interview and diagnostic test with teachers	August – September 2024

This study explores information obtained from human participants and non-human sources. Using these two data sources, the researcher aims to investigate the zone of concept image differences that occur in the internal transposition based on the theory of didactic transposition.

The researcher analyses information obtained from non-human participants to explore the concept image of linear equation in one variable from the perspective of knowledge to be taught. The study object is a seventh-grade mathematics textbook used in Indonesian middle schools. To enhance the credibility of the research, the textbooks selected were those that have been reviewed and published by the ministry of education, culture, research and technology for use as official learning resources in schools. The textbooks are widely used across Indonesian schools, ensuring that the selected materials represent the curriculum and educational standards followed by a majority of students in Indonesia. The section analyzed in the textbook is a series of tasks used in early algebra learning, specifically on linear equations in one variable.

The researcher analyzes information obtained from human participants to explore the concept image of linear equations in one variable from the perspective of taught knowledge. The subjects of this study are four mathematics teachers from Indonesian middle schools in Luwuk, Indonesia. Luwuk was chosen as the research site primarily to avoid language barriers during observations and in-depth interviews, which are crucial in capturing teachers' concept images through didactic praxeology. Conducting the study in Luwuk ensured participants felt comfortable using everyday language, enabling researchers to clearly understand and accurately interpret teachers' perspectives. Moreover, the selection of subjects was based on the consideration that seventh-grade students are currently studying the topic of linear equations in one variable. The subjects were chosen using purposive sampling. Although the sample size is small, it is adequate for this qualitative study as data saturation was achieved. After conducting interviews and analyzing the data, no new themes or insights emerged from additional data collection, indicating that the data was sufficient to capture the main variations and patterns in the concept images held by the teachers. The criteria for selecting teachers as research subjects included having a minimum of five years of teaching experience and their familiarity with the *Merdeka* curriculum. Selecting experienced teachers contributes significantly to confirmability in qualitative research, as their extensive practical knowledge provides rich and accurate descriptions of their concept images. This, in turn, supports the study’s objectives by ensuring a credible analysis of the interaction between scholarly knowledge, textbooks and teachers' instructional practices regarding linear equations in one variable.

The instruments used in this study consist of primary and supporting instruments. The primary instrument in this research is the researcher themselves. The credibility of the primary instrument is tested through Focus Group Discussions (FGD). The FGD, involved three distinguished experts: an algebra specialist, a mathematics education expert and a curriculum design authority, was conducted by presenting the findings of the task analysis, followed by structured discussions and critical feedback. This FGD was conducted to critically review and validate the findings of the task analysis, enhancing the credibility, verifiability and interpretability of the research outcomes. However, the supporting instruments are used to collect data and are divided into two types: test instruments and non-test instruments. The test instrument in this research is a diagnostic assessment of the topic of linear equations in one variable, designed to identify the concept image of the teachers. In designing the research instrument, the researcher referred to the results of the textbook analysis. There are three indicators used in this study to capture the teachers' concept image, which are: 1) Understanding the definition of a linear equation in one variable, 2) Modeling mathematical problems into equation form, and 3) Solving contextual problems using the properties of equations. The non-test instruments include interview guidelines with the teachers to justify the responses from the diagnostic test.

To explore the relationship between concept images from various institutions, the researcher used the Atlas.ti application as a tool to assist in the analysis. Atlas.ti was specifically utilized for coding qualitative data, identifying themes, and visualizing patterns and relationships among concept images from scholarly knowledge, textbooks and teachers. To ensure the accuracy of the analysis, an expert in Atlas.ti was consulted to review and validate the data. After recoding and examining the data, no discrepancies were identified between the expert's coding and the researchers' coding, thereby confirming the reliability and accuracy of the data for the thematic analysis.

To study the concept image from scholarly knowledge – knowledge to be taught – taught knowledge, the researcher uses the stages of qualitative research according to Ricoeur (1975) , which are: (1) Explanation: After the data is collected, the researcher transcribes, analyzes and summarizes the scholarly knowledge based on document analysis, conducts an analysis of the learning material on linear equations in one variable and analyzes the results of in-depth interviews with teachers; (2) Naïve understanding: The researcher develops notes and raw research data, details the main aspects of the learning process, describes what happens during the learning activities and what the teacher teaches during the process; (3) In-depth understanding: This stage involves analyzing and reinterpreting the relationships between the descriptions obtained to better understand the processes and causes of the formation of the concept image; (4) Appropriation: This stage involves comprehensively analyzing and interpreting the data obtained from mathematicians, textbooks and teachers, along with relevant theories, to ultimately identify differences and the causes of differences in the concept image of linear equations in one variable and to conclude the extent of the concept image gap that occurs in different institutions. ATD guided the analysis of how mathematical knowledge was transformed across different educational institutions (scholarly knowledge, textbooks, teachers), while concept image theory provided insights into the mental representations influencing these transformations.

Ricoeur's (1975) qualitative research stages were chosen due to their structured yet interpretative nature. Unlike other qualitative approaches, Ricoeur's framework facilitates iterative interpretation between explanation and deep understanding, allowing nuanced exploration of complex conceptual interactions. However, its interpretative flexibility requires rigorous reflexivity to minimize researcher bias. Figure 6 illustrate the research procedures to provide a step-by-step overview of the research process.

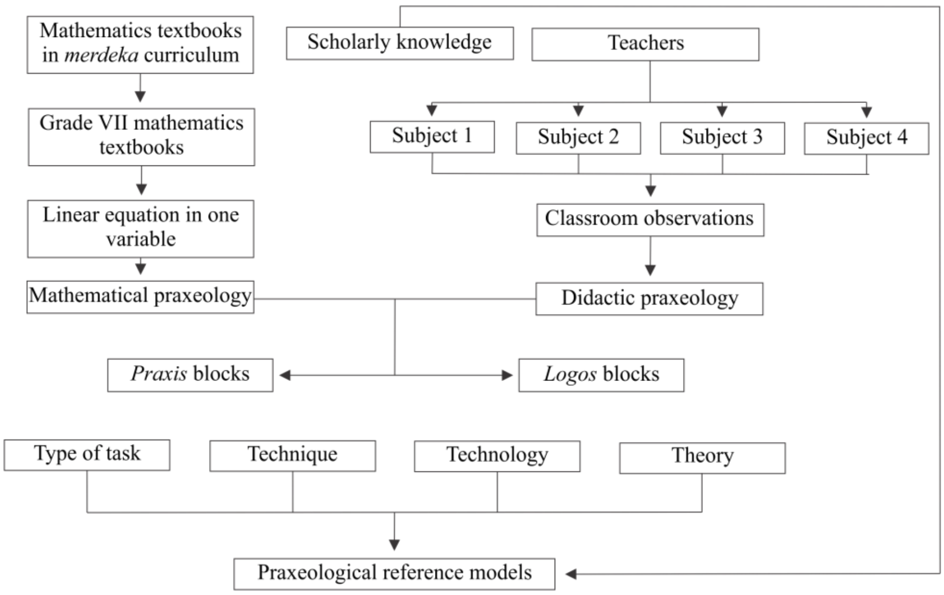

Figure 6. Research procedures

RESULTS AND DISCUSSION

Based on the research findings, several unexpected findings were identified, one of which is that scholarly knowledge, which is context-free, encounters challenges when contextualized. Scholarly knowledge, being abstract and theoretical, cannot be directly applied in a classroom context without undergoing modifications or adjustments. This was identified by the differences in the definition of linear equations between the scholarly knowledge version and how it was transposed into knowledge to be taught, taught knowledge, and learned knowledge.

Concept Image Differences in Knowledge to Be Taught (Textbooks)

One of the mathematics topics studied by students in Grade VII of junior high school (SMP) in Indonesia is linear equations in one variable. Figure 5 explains the definition of an equation according to the mathematics textbook of the Merdeka curriculum in Indonesia.

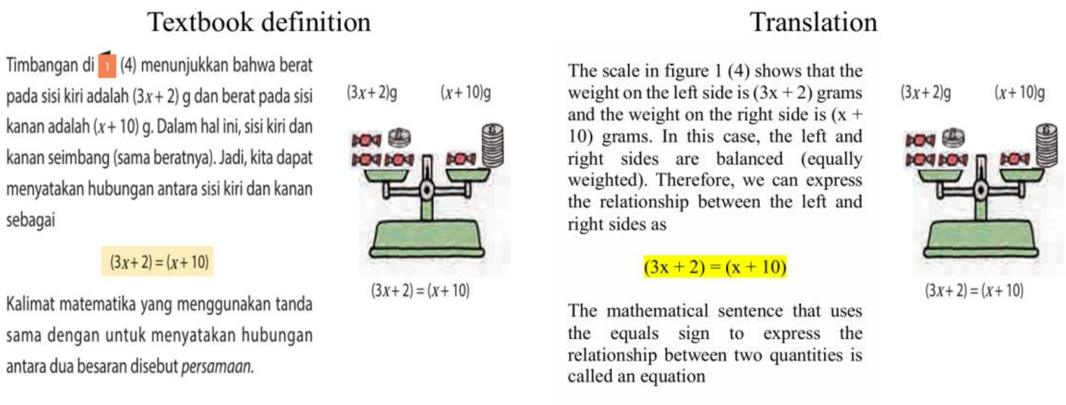

Figure 5. Textbooks definition of an equation

According to the Indonesian mathematics textbook in Figure 5, an equation is a mathematical statement that uses the equals sign (=) to indicate a relationship between two quantities ($C_1 - TB$). To facilitate the analysis of information using Atlas.ti, the researcher coded this concept image as $C_1 - TB$. The label "$C_1 - TB$" represents concept image 1 (C_1), which is found in textbooks (TB). This definition needs to be revised to explore the meaning of an equation, as it only emphasizes the use of the equals sign (=) without further explaining the role and properties of equations in mathematics. This definition might also make it difficult for students to distinguish between an equation and an identity (Berman & Shvartsman, 2016). The definitions provided in textbooks can potentially lead to misconceptions among students because they primarily focus on procedural aspects, such as using the equal sign merely as a signal to calculate results, rather than emphasizing its conceptual meaning as a symbol of equivalence (Ancheta & Subia, 2020; Johannes & Davenport, 2019; Ralston & Li, 2022). Figure 6 illustrates the definition of variable in the textbooks.

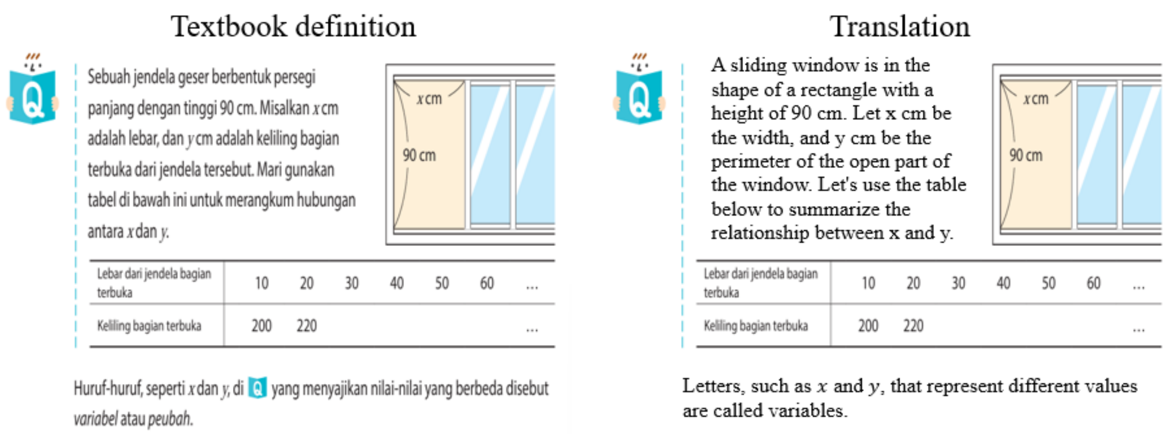

Figure 6. Textbooks definition of a variable

In Figure 6, the Indonesian mathematics textbook (*Merdeka* curriculum) defines a variable as an unknown value ($C_2 - TB$). Meanwhile, a coefficient is the value accompanying the variable ($C_3 - TB$), and a constant is a value that does not change ($C_4 - TB$). A praxeological analysis is conducted to understand the interconnectedness of the material on linear equations in one variable, both structurally and functionally. The numbering of these praxeological elements follows a systematic order, where each element is assigned a number based on its position in the teaching sequence presented in the textbook. T_1, for example, represents the first task introduced in the textbook, which is the initial problem or situation that students encounter in relation to linear equations in one variable. By using the praxeology approach, we can identify elements such as types of tasks (T), techniques (τ), technology (θ), and theory (Θ), as illustrated in Table 2.

Table 2. *Praxis* and *Logos* blocks in mathematical praxeology

Type of Task	Technique	Technology	Theory
T_1 : Understanding the truth of mathematical statements in equations when letters are substituted with numbers.	τ_1 : Perceptual τ_3 : Operational τ_4: Algebraic	θ_1 : Uses the equal sign to express the relationship between two quantities, defining it as an equation	Θ_1 : An open mathematical sentence using the equal sign to express the relationship between two quantities called an equation
T_2 : Understanding the solution of an equation without substituting numbers into letters.	τ_1 : Perceptual τ_3 : Operational τ_4: Algebraic	θ_2 : Techniques employed to eliminate or remove variables in a system of linear equations with the aim of finding solutions to the system	
T_3 : Solving the equation using the properties of equations	τ_1 : Perceptual τ_4: Algebraic	θ_3 : Procedures or actions used to manipulate algebraic expressions	Θ_2: If m is added, subtracted, multiplied, or divided on both sides, then the equation remains valid
T_4 : Solving equations using algebraic manipulation	τ_3 : Operational τ_4: Algebraic		
T_5 : Understanding a situation using linear equations	τ_3 : Operational τ_4: Algebraic		

Using the assistance of the Atlas.ti application, the researcher explores the relationships between each type of task (white node), technique (green node), technology (yellow node), and theory (red node) of praxeological analysis in the textbook, as presented in the Figure 7.

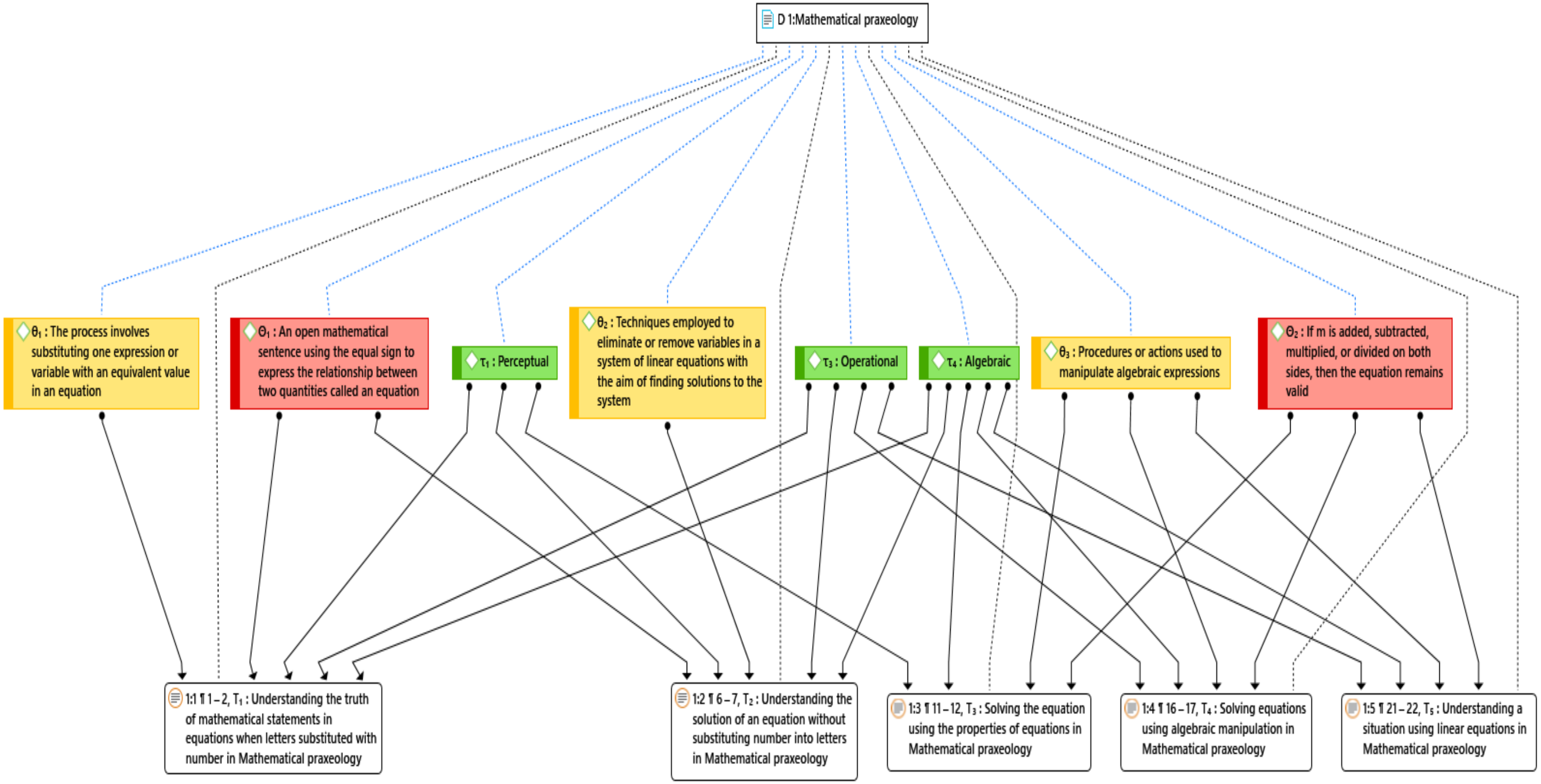

Figure 7. Relationship between T, τ, θ, Θ in praxeological analysis

Concept Image Differences in Taught Knowledge (Teachers)

Based on classroom learning observations, teachers possess various concept images regarding the definition of linear equations, as described in Table 3. To facilitate the analysis of information using Atlas.ti, the researcher coded the concept image as $C_{12} - TC$. The label "$C_{12} - TC$" represents concept image 1 from subject 2 (C_{12}), which is found in teacher perspective (TC).

Table 3. Definition of linear equation in teacher perspective

Subject	Definition	The general form
Subject 1	• An equation is a mathematical expression that expresses equality between two expressions or values ($C_{11} - TC$). • A variable is a symbol that can be replaced by a value, so there are many possible values that can be substituted into that symbol ($C_{21} - TC$). • A coefficient is a complementary number to a variable ($C_{31} - TC$). • A constant is a value that does not have a variable ($C_{41} - TC$).	A linear equation in one variable is an equation on the form: $ax + b = 0$ Where: a : Coefficient ($a \in$ R, a ≠ 0) b : Constant ($b \in$ R) x : Variable ($x \in$ R)
Subject 2	• An equation is a mathematical statement that expresses equality between two expressions or values, typically using the sign (=) ($C_{12} - TC$) • A variable is something whose value is sought ($C_{22} - TC$). • A coefficient is a value that appears in front of a variable ($C_{32} - TC$). • A constant is a value that does not have a variable ($C_{42} - TC$).	A linear equation in one variable is an equation on the form: $ax = b$ Where: a : Coefficient ($a \in$ R, a ≠ 0) b : Constant ($b \in$ R) x : Variable ($x \in$ R)
Subject 3	• An equation is an open sentence that has the equal sign (=) ($C_{13} - TC$). • A variable is something whose value is sought ($C_{23} - TC$). • A coefficient is a value adjacent to a variable ($C_{33} - TC$). • A constant is a value that does not have a variable ($C_{43} - TC$).	A linear equation in one variable is an equation on the form: $ax = b$ Where: a : Coefficient ($a \in$ R, a ≠ 0) b : Constant ($b \in$ R) x : Variable ($x \in$ R)
Subject 4	• An equation is a statement that expresses equality between two expressions or sums, typically using the equal sign (=) ($C_{14} - TC$) • A variable is a variable that needs to be found ($C_{24} - TC$). • A coefficient is a clear numerical value that is adjacent to a variable ($C_{34} - TC$). • A constant is something with a clear value ($C_{44} - TC$).	A linear equation in one variable is an equation on the form: $ax = b$ Where: a : Coefficient ($a \in$ R, a ≠ 0) b : Constant ($b \in$ R) x : Variable ($x \in$ R)

Based on Table 3, generally, teachers' conceptions of variables, coefficients and constants align with the definitions in textbooks and scholarly knowledge. However, discrepancies arise when teachers define an equation, with variations in their interpretations compared to formal definitions. These concept image differences can influence how teachers understand and teach algebraic concepts (Prihandhika et al., 2020). The potential consequences for student learning include confusion and misconceptions, especially if teachers fail to emphasize the concept of equivalence in equations (Kusuma et al., 2018). This can lead to procedural errors and hinder students' ability to grasp more advanced algebraic concepts.

Didactic praxeological analysis is conducted to further explore teachers' pedagogical abilities. This approach is used to understand teachers' teaching practices in the context of mathematics education. This analysis involves researching the interactions between the teacher, students and instructional materials in the classroom. Table 4 illustrates the *praxis* and *logos* blocks in didactic praxeological analysis.

Table 4. *Praxis* and *Logos* blocks in didactic praxeology

Type of Task	Technique	Technology	Theory
T_1 : Solving linear equations using the method of addition and subtraction	τ_2 : Physical τ_3 : Operational	θ_2 : Techniques employed to eliminate or remove variables in a system of linear equations with the aim of finding solutions to the system	Θ_1: An open mathematical sentence using the equal sign to express the relationship between two quantities called an equation
T_2 : Solving linear equations using the method of multiplication and division	τ_2 : Physical τ_3 : Operational τ_4 : Algebraic		
T_3 : Solving equations using the properties of equations	τ_2 : Physical τ_4 : Algebraic		Θ_2: If m is added, subtracted, multiplied, or divided on both sides, then the equation remains valid
T_4 : Solving equations using the idea of moving terms	τ_2 : Physical τ_4 : Algebraic	θ_3 : Procedures or actions used to manipulate algebraic expressions	
T_5 : Solving equations using parentheses	τ_2 : Physical τ_4 : Algebraic		
T_6 : Solving equations in decimal and fraction forms	τ_2 : Physical τ_3 : Operational		

Based on the analysis of the *praxis* block in didactic praxeology (Table 4), it was identified that teachers frequently use not only physical techniques (such as manipulatives) but also perceptual techniques (e.g., visual aids or diagrams) and operational techniques (e.g., step-by-step procedural instructions) to teach linear equations. These techniques align with the *Merdeka* curriculum's emphasis on active learning and making abstract concepts more accessible (Baharuddin & Setialaksana, 2023; Saa, 2024). While these methods are effective for helping students engage with the material, overreliance on them may hinder the development of abstract algebraic reasoning. For example, while beneficial for initial understanding, an overreliance on physical techniques may limit students' ability to transition to more abstract algebraic reasoning. This dependence could hinder their grasp of the underlying concepts when manipulatives are no longer used, affecting their ability to solve equations conceptually in more complex scenarios. Therefore, it is crucial to balance physical techniques with methods that foster deeper conceptual understanding (Fardian et al., 2025).

Using the assistance of the Atlas.ti application, the researcher explores the relationships between each type of task (white node), technique (green node), technology (yellow node), and theory (red node) in the textbook, as presented in the Figure 8.

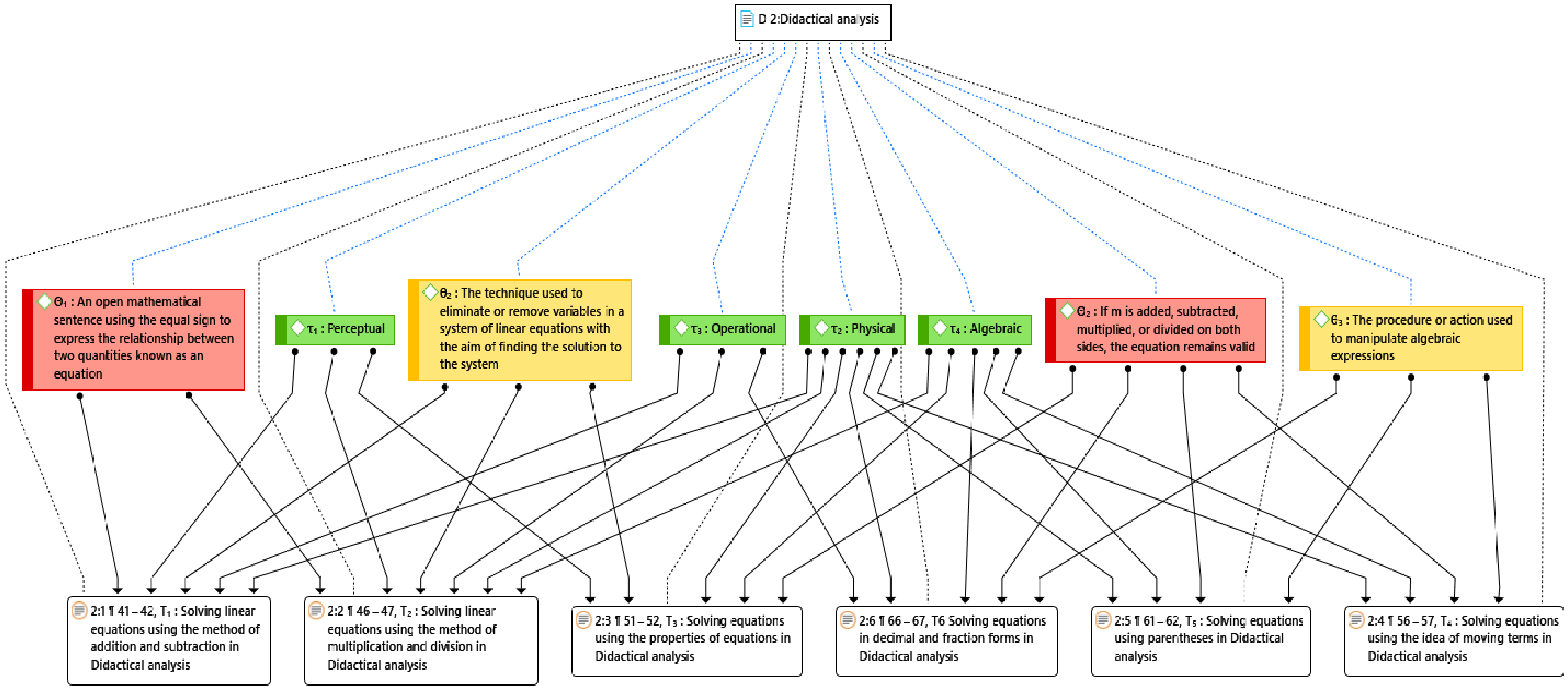

Figure 8. Relationship between T, τ, θ, Θ in didactical analysis

Zone of Concept Image Differences in Across Institution

In our study, we found that teachers in Indonesia often interpret the equal sign as an operator, prompting students to perform operations rather than seeing it as a symbol of equivalence. This interpretation aligns with findings from other countries, suggesting that this issue is not unique to Indonesia. Research from Jordan and India shows that students frequently see the equal sign as a prompt to perform an operation rather than as a symbol of equivalence (Eichhorn et al., 2018). Similarly, in South Africa, primary school students mostly interpret the equal sign operationally, which negatively impacts their performance in solving equations (McAuliffe et al., 2020). In the United States, this operational view of the equal sign is common among younger students and can persist into adulthood, affecting their overall mathematical performance (Fyfe et al., 2020). In contrast, studies from China and England show differing trends. Chinese students are more likely to understand the equal sign as indicating equivalence and substitution, while English students are less likely to adopt this interpretation (Jones et al., 2012). Similarly, in the United Arab Emirates (UAE), middle school students predominantly view the equal sign as a unidirectional symbol, used mainly to indicate the answer, similar to the operational view seen in other countries (Wardat et al., 2021).

These comparisons indicate that while the specific educational contexts vary, the misinterpretation of the equal sign is a widespread issue in mathematics education globally. The differences in concept images between scholarly knowledge, textbooks, and teachers' interpretations can potentially hinder students' mathematical development (Jamilah et al., 2020). When these concepts are inconsistently defined or taught, students can potentially develop misconceptions, limiting their understanding of core principles like equivalence in equations. This can lead to an overreliance on procedural knowledge rather than conceptual understanding, making it harder for students to apply mathematical concepts in new contexts. As a result, students could potentially struggle with more complex topics and fail to progress in their mathematical education, impeding their overall learning and problem-solving abilities. According to Brousseau (2002), this probllem can potentially cause didactic obstacles due to the mismatch of information taught by teachers, which stems from misconceptions about didactic design. This highlights the importance of addressing these misconceptions in teaching practices, both in Indonesia and internationally, to improve students' understanding of mathematical concepts. Figure 9 explores the concept images connection between each institution.

The codifications $C_1 - Tb$ and $C_{11} - TC$; $C_{12} - TC$; $C_{13} - TC$; $C_{14} - TC$ have a common understanding regarding the definition of equations. This mathematical statement uses the equal sign to express the relationship between two quantities. However, the concept image held by the knowledge to be taught and taught knowledge differs from the concept definition of mathematicians. When the definition of equations from scholarly knowledge is diffused to different institutions, such as knowledge to be taught and taught knowledge, a significant shift in meaning occurs. In the definition of equations according to scholarly knowledge, mathematicians emphasize the definition of equations as an "open sentence," but when it comes to knowledge to be taught and taught knowledge, the meaning of an "open sentence" is lost. Hence, equations only emphasize the "relationship between two quantities". Based on the interview results with the teacher, it was revealed that the teacher cannot differentiate between the definitions of "equation" and "equality".

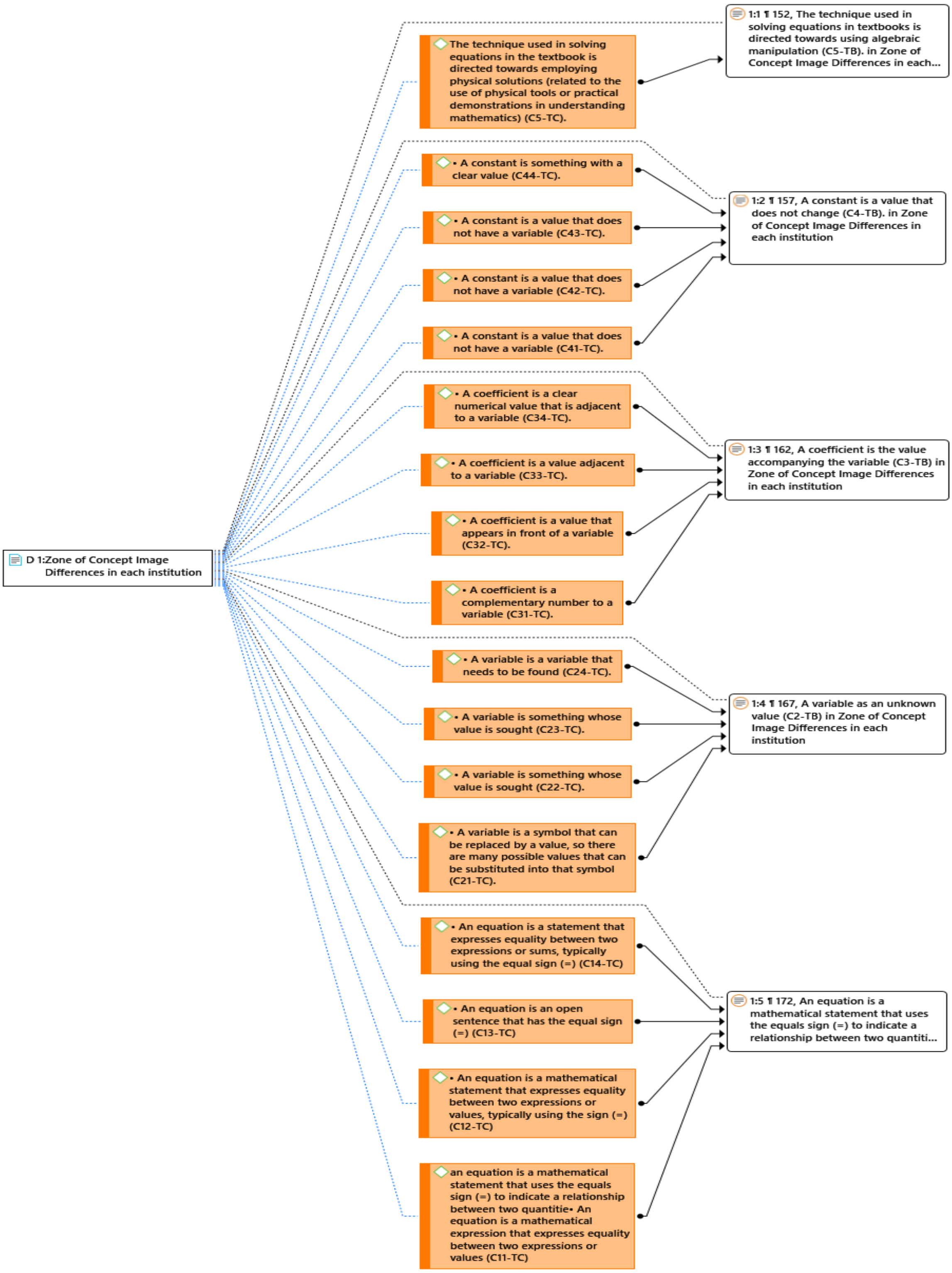

Figure 9. Zone of Concept Image Differences in each institution

Codifications $C_2 - TB$ dan $C_{21} - TC$; $C_{22} - TC$; $C_{23} - TC$; $C_{24} - TC$ have differences in interpretation regarding the concept of variables. Variables in the knowledge to be taught version are letters representing different values, while when transferred into taught knowledge, teachers interpret them as values to be sought for their truth. When knowledge from scholarly knowledge is diffused into knowledge to be taught, the meaning of variables does not change. However, the meaning changes when teachers attempt to transfer knowledge from this knowledge to be taught. Previously, the definition of variables was context-independent because it was a priori knowledge, but the definition of variables changes due to contextualization. Finally, mathematical knowledge transforms into a posteriori knowledge. Based on the interview results with the teacher, it is revealed that the teacher contextualizes to provide understanding to the students.

> Researcher: *How do you explain the concept of variables to students in the classroom learning process?*
> Subject 1: *To make students understand the concept of variables, I contextualize it so that students can understand it because it relates to their daily lives. I explained that a variable is a symbol representing an unknown value or something that can change. For example, when we calculate the total cost of shopping at the market, we may not know exactly how much each item we buy costs. In this case, we can use variables to represent the price of each item. I give concrete examples like: "Imagine we buy 2 apples and 3 oranges at the market. We symbolize apples with the variable* a *and oranges with the variable* j*. This way, students can see how variables are used to simplify and model real-life situations they face every day."*
> Researcher: *So, the variables are apples and oranges, right?*
> Subject 2: *Yes*

The mathematical knowledge that is context-independent can become problematic when contextualization is attempted (Hendriyanto et al., 2024; Suryadi, 2023). This can be seen from the concept image of the teacher who considers that the variables in the problem above are "oranges" and "apples", not "the price of oranges" and "the price of apples". In the given example, variables should represent the prices of apples and oranges, not the apples and oranges themselves. This is important because variables simplify and abstract real-life situations so students can model and solve problems more effectively. This mistake occurs due to attempts to link mathematical concepts with everyday life contexts without clearly explaining what the variables represent (Coştu et al., 2009). For example, suppose a teacher says that the variables are apples and oranges. In that case, students may become confused and assume that the variables are those physical objects, rather than the prices or quantities of those objects.

Codifications $C_3 - TB$ dan $C_{31} - TC$; $C_{32} - TC$; $C_{33} - TC$; $C_{34} - TC$ as well as $C_4 - TB$ dan $C_{41} - TC$; $C_{42} - TC$; $C_{43} - TC$; $C_{44} - TC$ have a common understanding of the concepts of coefficients and constants. However, the numbers accompanying the variables are called coefficients, while those not containing variables representing a number are called constants. Knowledge from scholarly knowledge and knowledge to be taught can be well absorbed by teachers in interpreting coefficients and constants. Coefficients are the numbers accompanying variables in a mathematical expression, indicating how many times the variable is multiplied. For example, in the expression $3x$, the number 3 is the coefficient of the variable x. Conversely, a constant is a standalone number without accompanying a variable in a mathematical expression. A constant is a fixed value that does not change.

Codifications $C_5 - TB$ and $C_5 - TC$ have differences in meaning related to the techniques used in solving linear equations. In the knowledge to be taught version, the five types of tasks based on the analysis of mathematical praxeology in the Grade VII mathematics textbook of the *Merdeka* curriculum predominantly use algebraic techniques. However, in the taught knowledge version, the six types of tasks found in didactic praxeology predominantly use physical techniques related to physical aids or practical demonstrations in understanding mathematics.

Implications for Mathematics Education in Indonesia

The description of the zone of concept image differences across different institutions highlights not only the variances in understanding linear equations but also points to the underlying causes of these differences. Figure 10 visualize the factors that cause differences in concept image from all institutions. In the diagram, the square shapes represent the institutions involved in the didactic transposition process, while the elliptical shapes represent the factors that contribute to the differences in concept images

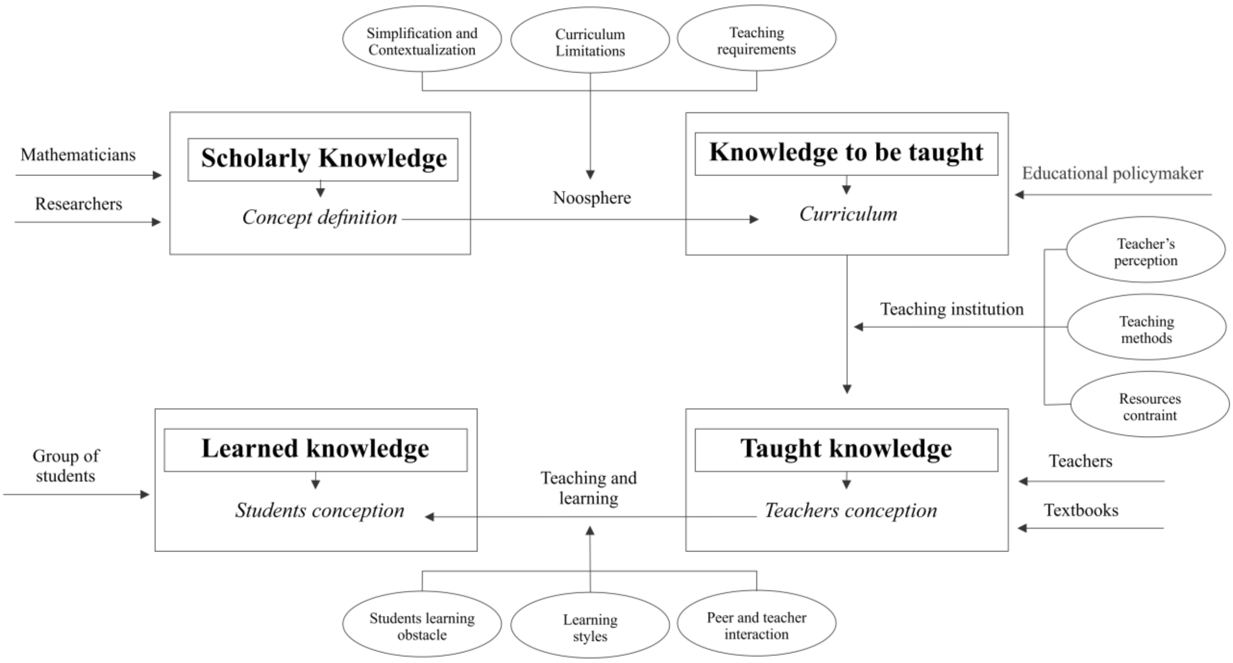

Figure 10. Causes of differences in concept image

The didactic transposition in Figure 10 explains how scholarly knowledge is simplified and adapted into teachable content. First, scholarly knowledge is transformed into knowledge to be taught, where complex ideas are made accessible for students through simplification. Next, knowledge to be taught becomes taught knowledge, as teachers interpret and present the material based on their methods and available resources. Finally, taught knowledge becomes learned knowledge, where students internalize and interpret the material, sometimes leading to misunderstandings due to prior knowledge or learning styles. This process creates gaps between theoretical knowledge and students' understanding in the classroom.

From the mathematical and didactic praxeological analysis, an alternative praxeological reference model for linear equations in the textbook has been formulated, visually depicted in Figure 11.

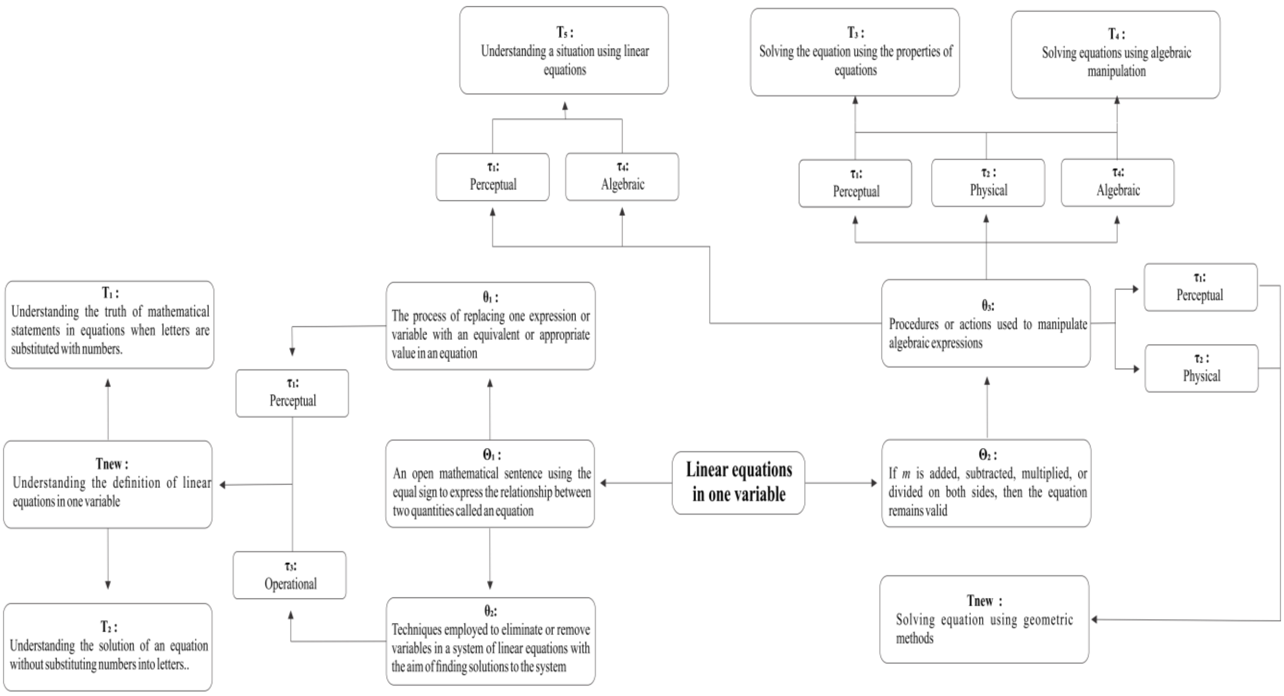

Figure 11. Praxeological reference models

Based on figure 11, the praxeological reference model presented in this study can serve as an alternative approach to teaching linear equations in one variable. The tasks within this model focus on six main issues related to linear equations. In the future curriculum, these tasks should be structured in a way that allows them to be solved using four distinct techniques: perceptual, physical, operational, and algebraic. Furthermore, there should be a redefinition of the equation, incorporating the term "open sentence" in accordance with scholarly knowledge. This will help ensure a more precise and accurate understanding of the concept of linear equations.

In Indonesia, curriculum changes are often influenced by political agendas, with shifts in leadership leading to changes in curriculum policies. This has resulted in the saying: “Change the minister, change the curriculum.” As the government transitions from President Joko Widodo to Prabowo Subianto in 2025, this shift will likely affect the direction and goals of the education system. This research has broader implications for Indonesia's educational future. The praxeological reference model, based on the transposition of knowledge from scholars to students, is well-suited for the new curriculum focused on deep learning. It bridges theoretical knowledge and student understanding, aligning with the curriculum's emphasis on internalizing concepts. However, challenges include the need for teacher training, curriculum adjustments and resource availability, especially in remote areas. There may also be resistance to adopting new teaching methods. Despite these challenges, the model offers an opportunity for meaningful educational reform in Indonesia.

CONCLUSION

This study identifies key differences in concept images across scholarly knowledge, textbooks and teacher knowledge, particularly regarding the topic of linear equations in one variable. These differences are significant as they highlight how mathematical knowledge is understood and transmitted at various levels, from scholars to teachers and students. However, this study also contributes to the existing theoretical framework in mathematics education by extending the work of Tall and Vinner on concept image and concept definition. Specifically, the findings show that the discrepancies in concept images, especially how teachers and textbooks interpret mathematical concepts, can influence students' understanding of these concepts. This aligns with Tall and Vinner’s theory that concept image and concept definition are crucial in shaping how mathematical knowledge is internalized. According to didactic transposition theory, knowledge undergoes a transformation as it moves from scholarly knowledge to the knowledge to be taught (textbooks) and finally to the taught knowledge by educators. The differences identified in this study reflect the challenges involved in this transposition, particularly in the case of linear equations, where the simplified or altered definitions in textbooks and teacher knowledge might not fully align with the theoretical definitions. This misalignment could potentially affect students’ conceptual understanding. These differences can potentially give rise to two types of learning obstacles: epistemological and didactical obstacles. Based on these findings, the researcher identified an observed concept image related to the equation concept, where teachers interpreted the equal sign in an equation as meaning “the answer is”. Teachers failed to understand the information provided in the mathematics textbook that the equal sign in the concept of a linear equation represents a quantitative equation, meaning the expression on the left side of the equal sign is equal to the expression on the right side of the equal sign.

Based on the findings of this study, it is recommended that textbooks be redesigned with a focus on presenting the five types of tasks based on the praxeological reference model, structured systematically to help students understand the concept of linear equations more comprehensively. The tasks should be designed to be solvable using the four techniques: perceptual, physical, operational, and algebraic. Additionally, there should be a redefinition of the equation, incorporating the phrase “open sentence” in line with scholarly knowledge to ensure a more accurate understanding of the concept. While this study provides valuable insights into the concept images of linear equations, there are several limitations that should be acknowledged. One limitation is the small sample size of four teachers, which may not fully represent the diversity of teaching practices and conceptions across different regions or educational levels in Indonesia. A larger sample would provide a more comprehensive view of variations in teachers' understanding. Additionally, the study primarily used qualitative methods, focusing on interviews and textbook analysis. Incorporating a mixed-methods approach, such as surveys and classroom observations, could enhance the robustness of the findings, allowing for more generalizable results and a deeper understanding of how concept images influence teaching and student learning. However, future research could focus on developing a didactic design based on the praxeological reference model proposed in this study. Future research could explore specific questions, such as how socio-cultural factors influence students' concept images or how teacher biases affect teaching practices. Additionally, interdisciplinary approaches, such as cognitive psychology and sociocultural theories, could provide deeper insights into the factors shaping students' concept images and learning experiences.

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AUTHOR CONTRIBUTIONS

DF: Writing – original draft, Conceptualization, Investigation, Resources, Visualization; DS: Writing – review and editing, Formal analysis, Methodology, Supervision, Validation; SP: Writing – original draft, Formal analysis, Methodology, Validation; AJ: Writing – review and editing, Methodology, Validation; SFA: Writing – review and editing, Validation.

CONFLICTS OF INTEREST

The author(s) declare no conflict of interest.

USE OF ARTIFICIAL INTELLIGENCE (AI)-ASSISTED TECHNOLOGY

The authors declare that no artificial intelligence (AI) tools were used in the generation, analysis, or writing of this manuscript. All aspects of the research, including data collection, interpretation, and manuscript preparation, were carried out entirely by the authors without the assistance of AI-based technologies.

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