OPEN ACCESS
Jurnal Ilmiah Ilmu Terapan Universitas Jambi
Vol. 10 No. 2. April 2026, pp. 1126-1137
DOI. 10.22437/jiituj.v10i2.53849
Review Article

PORTFOLIO OPTIMIZATION UNDER CARDINALITY CONSTRAINTS: A METAHEURISTIC MEAN-VARIANCE APPROACH

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Article Info
Received: Mar 10, 2026
Revised: Mar 30, 2026
Accepted: Apr 22, 2026
OnlineVersion: Apr 30, 2026

Abstract
The growing emphasis on sustainability in investment decisions necessitates portfolio optimization models that incorporate practical constraints, particularly asset cardinality. This study applies a Cardinality-Constrained Mean–Variance (CCMV) framework, which increases computational complexity due to the limited number of selected assets. To address this, two metaheuristic algorithms, namely Particle Swarm Optimization (PSO) and Artificial Bee Colony (ABC) are implemented. Using data from 25 stocks observed between March and August 2025, the analysis includes asset screening based on the risk-free rate, portfolio optimization, and performance evaluation using expected return, risk, and the Sharpe ratio. PSO produces an optimal portfolio of 8 assets with a Sharpe ratio of 27.594%, while ABC selects 9 assets and achieves a slightly higher Sharpe ratio of 27.599%. Although the difference is marginal, ABC demonstrates superior computational efficiency. The findings highlight the effectiveness of metaheuristic approaches, particularly ABC, in solving constrained portfolio optimization problems.

Keywords: Artificial Bee Colony, Cardinality Constrained Mean Variance, Metaheuristic, Particle Swarm Optimization.

INTRODUCTION

Despite the rapid growth of cybersecurity behavior research, the existing literature remains conceptually fragmented. While Protection Motivation Theory (PMT) has been widely used to explain individual security behavior, prior studies tend to focus on isolated psychological determinants such as threat perception, self-efficacy, and awareness, without providing a comprehensive integration of these constructs (Sommestad & Hallberg, 2015; Kiran et al., 2025a).

The capital market plays a crucial role in promoting investment and supporting economic growth by channeling funds from investors to issuers (Ahmed & Chowdhury, 2024). In recent years, investor participation has trended upward, reflecting growing confidence in capital markets and long-term investment opportunities. This positive trend has been evident through the steady expansion of investor activity and engagement. Despite this positive trend, investors continue to face uncertainty regarding expected returns and associated risks. Return and risk are inherently linked, where higher risk is accompanied by higher expected return (Bagh et al., 2025). One of the primary strategies to manage such risk is portfolio diversification (Yaman & Tuncel, 2025).

Portfolio diversification involves constructing a portfolio by combining selected stocks to minimize risk without sacrificing expected return (Eom et al., 2021). Over time, diversification strategies have increasingly incorporated sustainability considerations. Environmental, Social, and Governance (ESG) principles have become central in investment decision-making (Sun, 2024; Yu et al., 2024). Moreover, Wang, (2025) argues that ESG-based investments tend to be more resilient during market volatility and offer better long-term performance potential. By integrating ESG principles, investors pursue not only financial returns but also broader social and environmental impacts. ESG-based indices have increasingly been used to represent the performance of sustainable investments and to serve as benchmarks for companies with strong environmental, social, and governance practices (Christine et al., 2025; Short & Ndlovu, 2025). Empirical evidence suggests that such indices tend to demonstrate relatively strong resilience, particularly during periods of market uncertainty. In recent years, ESG-oriented portfolios have delivered competitive returns relative to conventional benchmarks and exhibited greater stability during market downturns (Arouri et al., 2025). This performance pattern indicates that ESG-aligned assets often possess defensive characteristics, making them attractive for investors seeking long-term, risk-adjusted returns. The characteristics of ESG-based stocks underline the importance of applying an appropriate quantitative framework to construct an optimal portfolio. The Mean-Variance model developed by Markowitz (1952) remains a foundational approach, as it determines asset allocation that minimizes risk for a given return level. However, the classical model does not account for practical investment constraints, such as the limitation on the number of assets in a portfolio (González-Bueno et al., 2025; Mansouri & Moghadam, 2021; Pires et al., 2026; Taheripour et al., 2025). To address this limitation, Chang et al. (2000) introduced the Cardinality-Constrained Mean-Variance (CCMV) model, which incorporates constraints on the number of selected assets and minimum and maximum weight bounds. The inclusion of these constraints increases computational complexity, making metaheuristic algorithms a suitable solution due to their ability to efficiently explore complex search spaces (Febrianti et al., 2022; Fu et al., 2023; Kalayci et al., 2019).

Among various metaheuristic techniques, Particle Swarm Optimization (PSO) and Artificial Bee Colony (ABC) are widely recognized for solving complex optimization problems. PSO, introduced by Kennedy & Eberhart (n.d.), is valued for its fast convergence and high-quality solutions (Kulkarni & Desai, 2016). Additionally, Cura (2009) shows that PSO yields higher returns and lower risk than the Genetic Algorithm (GA), Simulated Annealing (SA), and Tabu Search (TS) in CCMV optimization. Furthermore, PSO achieves superior fitness values compared to Ant Colony Optimization (ACO) according to the researcher (Priyadarshi & Kumar, 2025). Meanwhile, the ABC algorithm, proposed by Karaboga (2005), mimics the foraging behavior of honeybee colonies and is recognized for its strong exploration capability and competitive solution quality (Dash et al., 2021; Karaboga, 2005). The ABC outperforms SA, TS, and Variable Neighborhood Search (VNS) in global exploration and solution quality within the CCMV framework (Chen et al., 2012). Nevertheless, (Kalayci et al., 2019) emphasizes that no single metaheuristic algorithm is universally superior under all conditions. Therefore, a direct comparison between PSO and ABC is necessary to determine the most suitable approach for ESG-based portfolio optimization within the CCMV framework.

Although both PSO and ABC have been extensively applied in portfolio optimization studies, little research has directly compared their performance in ESG-based portfolios. Optimization of ESG stocks without incorporating investment constraints or metaheuristic approaches was studied by Gasmara et al. (2023). Moreover, Müller & Joubrel (2025) applied the Mean-Variance model to ESG Leaders stocks but did not integrate complex constraint structures (López Prol & Kim, 2022; González-Bueno et al., 2025; León-Camacho et al., 2025; Wu, 2025). These gaps indicate that ESG portfolio construction methods remain insufficiently optimized in addressing dynamic market conditions. Accordingly, implementing the CCMV model, combined with metaheuristic algorithms, is essential to enhance the effectiveness of ESG-based portfolio management.

By integrating the CCMV model with PSO and ABC, this study aims to develop a portfolio optimization strategy that is both quantitatively efficient and aligned with sustainability principles in the Indonesian capital market context. The findings are expected to contribute to the sustainable finance literature and provide practical implications for institutional and retail investors through implementation in a dashboard-based system. Specifically, this research seeks to identify the most appropriate algorithm for constructing an optimal ESG-based portfolio that advances sustainable investment practices in response to evolving market conditions.

RESEARCH METHOD

This study employed a quantitative research design using a computational and optimization approach. The research focused on constructing an optimal portfolio using the Cardinality Constrained Mean-Variance (CCMV) model. The optimization process was carried out using two metaheuristic algorithms, namely Particle Swarm Optimization (PSO) and Artificial Bee Colony (ABC). The study aimed to compare the performance of the two algorithms in generating optimal portfolio compositions subject to cardinality constraints.

The observation period spanned from March 3, 2025, to August 29, 2025, referring to the official evaluation period of the ESG Index. The 25 companies analyzed were selected based on the ESG-based evaluation for the period June 2, 2025, to November 28, 2025. Daily closing price data for the index constituents were obtained from Yahoo Finance, a publicly accessible financial data source. The risk-free rate was proxied by the BI Rate, obtained directly from Bank Indonesia's official website. Data processing and portfolio optimization were performed using Python.

The research procedure was conducted sequentially as follows:

1. Collecting historical daily closing stock price data of issuers included in the ESG Index during the observation period using programmatic data retrieval through Python. The risk-free rate, represented by the BI Rate, was obtained from Bank Indonesia's official website.
2. Calculating stock returns and expected returns for each issuer based on the historical closing price data.
3. Conducting an initial screening process by comparing the expected return of each issuer with the risk-free rate. Issuers with expected returns below the risk-free rate were eliminated, while those that met the criterion were retained for further analysis.
4. Calculating the variance and covariance matrix of stock returns for issuers that passed the screening stage.
5. Formulating the Mean-Variance model and incorporating cardinality constraints along with lower and upper weight bounds to construct the Cardinality Constrained Mean-Variance (CCMV) model.
6. Implementing the Particle Swarm Optimization (PSO) and Artificial Bee Colony (ABC) algorithms to solve the CCMV model by generating portfolio weight allocations under stochastic search mechanisms.
7. Performing parameter exploration by varying the number of selected issuers (K) from 5 up to the maximum number of issuers that passed the screening stage and varying the risk preference parameter (λ) from 0 to 1 with an increment of 0.02.
8. Evaluating each generated portfolio using the Sharpe ratio to identify the optimal portfolio based on the best trade-off between return and risk.
9. Comparing the optimal portfolios obtained from PSO and ABC to determine the best-performing algorithm.
10. Developing a web-based dashboard prototype using the selected best optimization algorithm.
11. Interpreting the analysis results in accordance with the research objectives.
12. Drawing conclusions and providing recommendations based on the research findings.

The data used in this study were secondary time series, consisting of daily closing stock prices for issuers included in the ESG Index and the risk-free rate, represented by the BI Rate. The observation period followed the predetermined research timeframe, which was based on the index evaluation period. The stock price data were obtained from Yahoo Finance, while the risk-free rate data were obtained directly from the official website of Bank Indonesia. The data collection process was conducted using a programmatic data retrieval technique through the Python programming language. Historical stock price data were directly downloaded using a financial data library connected to Yahoo Finance, which allowed automated extraction of daily closing prices for each issuer. The retrieved data were then compiled into structured time-series datasets for further processing.

After the data were collected, preprocessing steps were carried out, including calculating stock returns and expected returns for each issuer. The expected return values were subsequently compared with the risk-free rate (BI Rate) to screen for outliers. Issuers with expected returns lower than the risk-free rate were eliminated and excluded from further analysis, while issuers that satisfied the criterion were retained for portfolio optimization.

The instruments used in this study were computational tools implemented in Python for data preprocessing, numerical computation, and optimization modeling. Python was used to implement the Cardinality-Constrained Mean-Variance (CCMV) model, as well as the Particle Swarm Optimization (PSO) and Artificial Bee Colony (ABC) algorithms. The optimization process involved iterative stochastic search to determine portfolio weight allocations under cardinality constraints. Parameter exploration was performed by varying the number of selected issuers (K) and the risk preference parameter (λ) within predefined ranges. The optimal portfolio was determined by the highest Sharpe ratio, and the best-performing algorithm was then used to develop the web-based dashboard prototype.

The data analysis in this study employed quantitative and computational approaches to construct an optimal portfolio using the Cardinality-Constrained Mean-Variance (CCMV) model and metaheuristic optimization algorithms. The analysis aimed to obtain the optimal combination of issuer weights that provides the best trade-off between return and risk in accordance with the research objectives.

The stages of data analysis were carried out sequentially as follows:

Calculating Return and Expected return, Calculating Realized Return, realized return aims to measure the rate of return obtained from stock investment, calculated as the difference in stock prices between two periods. Calculated using Equation 1.

$$\mathbf{R_{i,t}} = \frac{\mathbf{P_{i,t}} - \mathbf{P_{i,t-1}}}{\mathbf{P_{i,t-1}}} \qquad (\mathbf{1})$$

Expected return is the return anticipated in the future and is inherently uncertain. To estimate this value, the geometric mean method is used as it accounts for the compounding effect over time. Calculated using Equation 2.

$$\boldsymbol{E(R_i)} = \sqrt[T]{\boldsymbol{\Pi_{t=1}^{T}\left(1 + R_{i,t}\right)}} - \mathbf{1} \qquad (\mathbf{2})$$

Screening Based on Risk-Free Rate, the expected return of each issuer was compared with the risk-free rate represented by the BI Rate. Issuers with expected returns lower than the risk-free rate were eliminated, while the remaining issuers were retained as inputs for the optimization process. The RFR value used is the BI Rate as of August 2025, which is 5%, calculated using equation 3.

$$\mathbf{RFR} = \frac{\mathbf{5\%}}{\mathbf{365}} = \mathbf{0.014\%} \qquad (\mathbf{3})$$

Risk is the level of return uncertainty relative to the expected return, measured using standard deviation to show the magnitude of return deviation from its expected value (Anderson et al., 2009). Calculated using Equation 4.

$$\boldsymbol{\hat{\sigma}_i} = \sqrt{\frac{\boldsymbol{\sum_{t=1}^{T}\left(R_{i,t} - E(R_i)\right)^2}}{\boldsymbol{T - 1}}} \qquad (\mathbf{4})$$

Calculating Covariance. Covariance indicates the directional relationship of return movements between two stocks (Li et al., 2026). A positive value shows two stocks moving in the same direction, while a negative value shows them moving in opposite directions. Calculated using Equation 5.

$$\hat{\sigma}_{ij} = \frac{\sum_{t=1}^{T}[R_{i,t} - E(R_i)][R_{j,t} - E(R_j)]}{T - 1} \qquad (5)$$

Formulation of the Cardinality Constrained Mean-Variance (CCMV) Model, the objective function in this CCMV model is designed to minimize risk and maximize returns, with the parameter $\boldsymbol{\lambda}$ determining the weight between the two. A $\boldsymbol{\lambda}$ value close to 1 emphasizes risk minimization, while $\boldsymbol{\lambda}$ close to 0 emphasizes returns. By varying $\boldsymbol{\lambda}$ within the range $\mathbf{0} \leq \boldsymbol{\lambda} \leq \mathbf{1}$, the entire efficient frontier can be traced. The objective function and its constraints are in Equation 6.

$$f_k^y = Min\, \lambda \left[\sum_{i=1}^{N}\sum_{j=1}^{N} x_i^y x_j^y \hat{\sigma}_{ij}\right] - (1 - \lambda)\left[\sum_{i=1}^{N} x_i^y \mu_i\right] \qquad (6)$$

Subject to the following constraints.

$$\sum_{i=1}^{N} x_i^y = 1 \tag{7}$$

$$\sum_{i=1}^{N} z_i = K \tag{8}$$

$$\varepsilon_i z_i \leq x_i^y \leq \delta_i z_i;\ i = 1, \dots, N \tag{9}$$

$$z_i \in \{0,1\};\ i = 1, \dots, N \tag{10}$$

The first constraint, as shown in Equation 7, is the budget constraint which ensures that the total proportion of weights across all selected assets equals exactly 1, meaning all available funds are fully invested. The second constraint, defined in Equation 8, is the cardinality constraint that limits the number of assets in the portfolio to exactly *K*. Furthermore, Equation 9 establishes the floor and ceiling constraints (buy-in constraints), ensuring that the weight of each selected asset stays within the predefined lower bound ($\boldsymbol{\varepsilon_i}$ = 0.01) and upper bound ($\boldsymbol{\delta_i}$ = 1). This prevents the model from assigning negligible proportions to assets. Finally, Equation 10 introduces the binary decision variable ($\boldsymbol{z_i}$), which indicates whether a stock is included in the portfolio ($\boldsymbol{z_i}$ = 1) or excluded ($\boldsymbol{z_i}$ = 0), thereby linking the stock selection process to the weight allocation.

1. Portfolio Optimization Using Metaheuristic Algorithms. The optimization process was carried out using Particle Swarm Optimization (PSO) and Artificial Bee Colony (ABC) algorithms. Both algorithms applied stochastic search mechanisms through random initialization and iterative updates to generate candidate portfolio weights that satisfy the defined constraints.
2. Portfolio Performance Evaluation. Portfolio performance is evaluated to determine the effectiveness of the asset allocation in achieving the optimal balance between return and risk. This study utilizes three primary indicators to measure the quality of the resulting portfolios.

Portfolio Expected Return is calculated as the weighted average of the expected returns of individual stocks within the portfolio. Where, $E(R_p)$ denotes the expected return of the portfolio, $E(R_i)$ represents the expected return of stock *i*, and x_{ki}^y is the weight of stock i at iteration *y* in solution *k*. The portfolio expected return is calculated using Equation 11.

$$E(R_p) = \sum_{i=1}^{N} \left[x_{ki}^y . E(R_i)\right] \tag{11}$$

Portfolio Risk is measured by the standard deviation of the portfolio (Lorimer et al., 2024), which accounts for the variances of individual assets and the covariances between asset pairs based on their respective weights. Where, $\hat{\sigma}_p$ represents portfolio risk, $\hat{\sigma}_{ij}$ denotes the covariance between stock i and stock j, and x_{ki}^y is weight of stock i at iteration *y* in solution *k*. Calculated using Equation 12.

$$\hat{\sigma}_p = \sqrt{\sum_{i=1}^{N} \sum_{j=1}^{N} x_{ki}^y x_{kj}^y \hat{\sigma}_{ij}} \tag{12}$$

Sharpe Rasio (Kourtis, 2016), The principal metric used to evaluate risk-adjusted performance. It measures the excess return of the portfolio relative to the risk-free rate per unit of total risk. A higher Sharpe ratio indicates a more efficient portfolio that provides better returns for the risk taken. Where, $E(R_p)$ is expected return of portfolio and $\hat{\sigma}_p$ is portfolio risk. Calculated using Equation 13.

$$Sharpe = \frac{E(R_p) - RFR}{\hat{\sigma}_p} \tag{13}$$

RESULTS AND DISCUSSION

Optimal Portfolio Optimization on the ESG Index Using PSO and ABC

The portfolio construction process in this study follows a sequence of analytical stages to identify the optimal ESG-based stock portfolio. The analysis begins with a stock elimination stage, followed by portfolio optimization using two metaheuristic algorithms, namely Particle Swarm Optimization (PSO) and Artificial Bee Colony (ABC). The performance of each algorithm is then evaluated and compared to identify the most effective approach for constructing the optimal portfolio within the Cardinality-Constrained Mean-Variance framework.

Stock Elimination Based on Risk-Free Rate

This subsection explains the stock elimination process based on the comparison between individual stock returns and the risk-free rate, ensure that only eligible stocks are considered in the subsequent portfolio optimization stage.

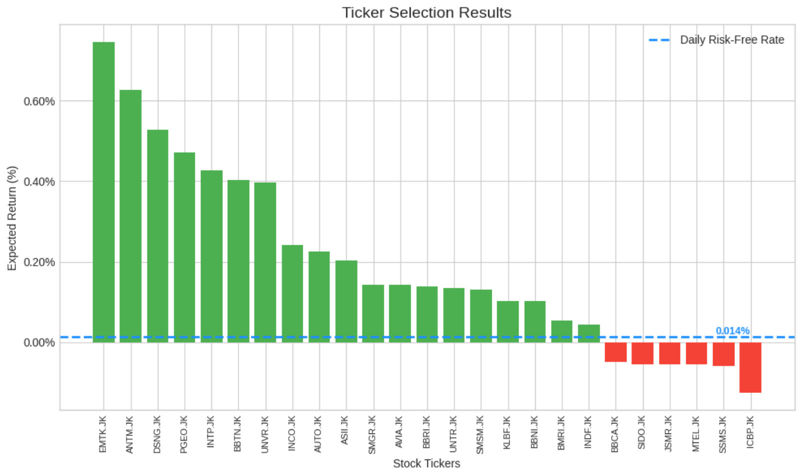

Figure 1. Expected Return of ESG-based Issuers

Based on Figure 1, the stock selection process was conducted on 25 issuers listed in the ESG Index for the period of March to August 2025. The screening was performed by comparing each issuer's daily expected return with a daily risk-free rate (RFR) of 0.014%. This stage aims to filter out issuers capable of providing returns above the risk-free asset to ensure a more effective optimization process. The selection results identified 19 issuers that passed the criteria, namely: AALI.JK, ADRO.JK, AMRT.JK, ASII.JK, BBNI.JK, BBRI.JK, BMRI.JK, BSDE.JK, CPIN.JK, EXCL.JK, INDF.JK, INTP.JK, ITMG.JK, KLBF.JK, PGAS.JK, PTBA.JK, SMGR.JK, SMSM.JK, and UNTR.JK. Conversely, six issuers were eliminated as their expected returns were below or equal to the RFR, specifically: BBCA.JK, ICBP.JK, JSMR.JK, MTEL.JK, SIDO.JK, and SSMS.JK. The issuers that passed this screening were subsequently used as inputs to construct the optimal portfolio using the Cardinality-Constrained Mean-Variance (CCMV) model. The descriptive statistical analysis results for the issuers that passed the screening process are presented in Table 1.

Table 1. Descriptive Statistics of Returns of Issuers

Issuer	*Mean*	Standar Deviation	Minimum	Maximum
ANTM.JK	0.627%	3.861%	-14.373%	10.508%
INCO.JK	0.241%	4.227%	-14.978%	12.628%
ASII.JK	0.203%	2.048%	-8.943%	9.950%
INDF.JK	0.043%	2.084%	-7.292%	6.667%
AUTO.JK	0.225%	1.895%	-8.354%	6.957%
INTP.JK	0.426%	3.212%	-12.692%	9.804%
AVIA.JK	0.142%	1.841%	-4.661%	7.143%
KLBF.JK	0.102%	2.991%	-7.048%	8.333%
BBNI.JK	0.101%	2.400%	-7.598%	8.974%
BBRI.JK	0.138%	2.359%	-10.123%	6.510%
BBTN.JK	0.403%	3.175%	-8.475%	9.953%
SMGR.JK	0.143%	3.753%	-14.717%	14.953%
BMRI.JK	0.054%	2.454%	-10.192%	8.650%
SMSM.JK	0.131%	1.586%	-5.276%	3.342%
DSNG.JK	0.527%	3.334%	-7.237%	12.727%

Issuer	*Mean*	Standar Deviation	Minimum	Maximum
EMTK.JK	0.745%	4.323%	-11.560%	17.949%
UNTR.JK	0.135%	2.283%	-14.650%	6.764%
UNVR.JK	0.397%	3.436%	-6.719%	17.057%
PGEO.JK	0.472%	4.052%	-6.831%	17.593%

Table 1. presents the descriptive statistics for the daily returns of the selected issuers. The results show that EMTK.JK, ANTM.JK, and DSNG.JK achieved the highest average returns at 0.743%, 0.627%, and 0.527%, respectively, accompanied by high standard deviations of 4.137%, 3.738%, and 3.234%. These high standard deviations indicate that the returns of these three stocks fluctuate significantly from their averages, as evidenced by EMTK.JK's wide range between -11.561% and 17.949%. While these high-risk, high-return stocks can enhance the portfolio's expected return, they also increase the overall risk. Consequently, more stable assets like SMSM.JK which exhibits a low standard deviation of 1.312% and a narrower fluctuation range from -5.276% to 3.342% are essential for risk mitigation. The strategic combination of high-growth assets and stable issuers is expected to facilitate the construction of an optimal portfolio through effective diversification.

Portfolio Optimization Using Particle Swarm Optimization (PSO)

This section discusses the formation of an optimal portfolio using the Particle Swarm Optimization (PSO) algorithm, including the determination of the optimal number of stocks (*K*) in the portfolio, weight allocation, and the visualization of the PSO algorithm's efficient frontier, as explained below. The movement of the Sharpe ratio for each *K* value is illustrated in Figure 2.

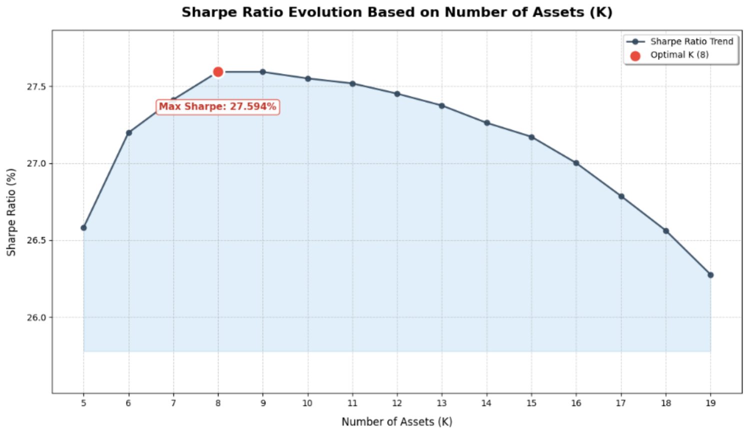

Figure 2. Sharpe Ratio Evolution of PSO

Figure 2. illustrates the trend of the Sharpe ratio relative to the number of assets (*K*) in the portfolio. Initially, the Sharpe ratio increases with *K* as asset diversification effectively reduces portfolio risk. However, the ratio plateaus after reaching an optimal threshold, as additional stocks exhibit high covariance with existing holdings, thereby increasing overall risk without providing further diversification benefits. Among the tested configurations, the optimal portfolio was identified at K=8 with a risk-weighting parameter (λ) of 0.90, yielding the highest Sharpe ratio of 27.594%. This Sharpe value indicates that the portfolio can generate an excess return of 27.594% relative to the total risk undertaken, outperforming all other asset combinations within the ESG Index.

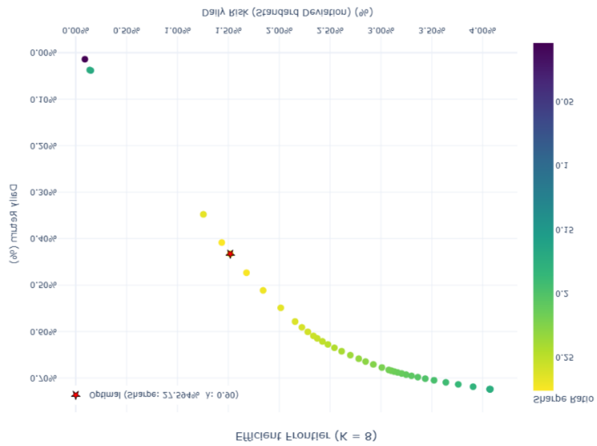

Figure 3. Efficient Frontier of PSO

Figure 3. illustrates the efficient frontier for the portfolio with K=8 by varying the risk-weighting parameter (λ) from 0 to 1. Each point on the curve depicts a specific risk-return combination, with the color gradient reflecting the Sharpe ratio levels. The optimal portfolio, identified by a red star, is achieved at λ=0.90, yielding the maximum Sharpe ratio of 27.594%. This signifies that the configuration provides the superior risk-adjusted return compared to all other tested parameter variations within the PSO algorithm. The specific weight allocation for each selected asset within this optimal portfolio is presented in Figure 4.

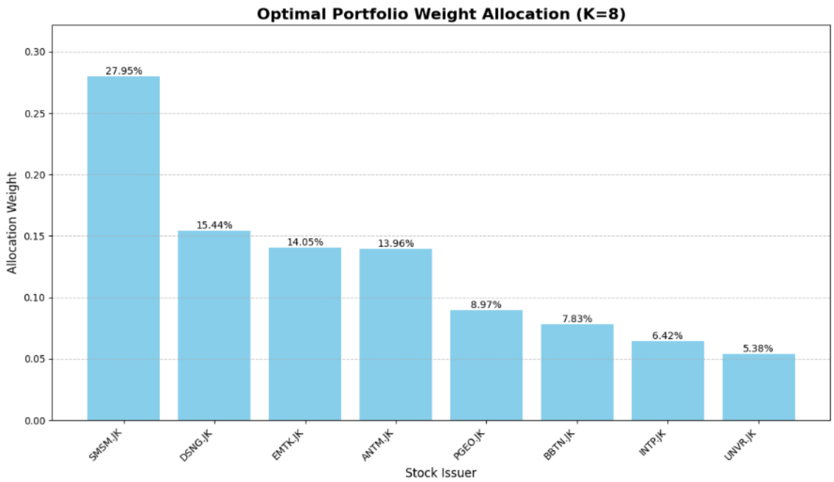

Figure 4. Portfolio Allocation using PSO

Figure 4 shows the optimal portfolio generated by the PSO algorithm (K = 8) exhibits a moderately concentrated allocation pattern. One dominant stock account for approximately 27.95% of the total allocation, indicating that PSO strongly prioritizes assets with superior risk–return characteristics. The second tier of assets receives relatively balanced weights of 13%–15%, reflecting their role in supporting return generation while maintaining diversification. The remaining assets are assigned progressively to smaller weights, ranging from 5% to 9%, suggesting their function as risk diversifiers rather than primary return drivers. PSO avoids extremely marginal allocations, resulting in a more even distribution among the lower-weight assets. This allocation structure highlights that PSO tends to produce a more compact and slightly more balanced portfolio, emphasizing efficiency in asset selection under cardinality constraints.

Portfolio Optimization Using Artificial Bee Colony (ABC)

This section discusses the formation of an optimal portfolio using the Artificial Bee Colony (ABC) algorithm, including the determination of the optimal number of stocks (K) in the portfolio, weight allocation, and the visualization of the ABC algorithm's efficient frontier, as explained below. The movement of the Sharpe ratio for each *K* value is illustrated in Figure 5.

Figure 5. illustrates the trend of the Sharpe ratio relative to the number of assets (*K*) using the ABC algorithm. The Sharpe ratio initially increases as *K* rises, reflecting the risk-reduction benefits of diversification. However, the ratio begins to plateau or decline after reaching an optimal threshold because additional stocks exhibit high covariance with the existing assets, thereby increasing portfolio risk without a corresponding increase in expected returns. The optimal configuration was identified at K=9 with a risk-weighting parameter (λ) of 0.90, achieving the maximum Sharpe ratio of 27.599%. This peak value represents an excess return of 27.599% per unit of daily risk, signifying the most efficient asset combination within the ESG Index for the ABC algorithm.

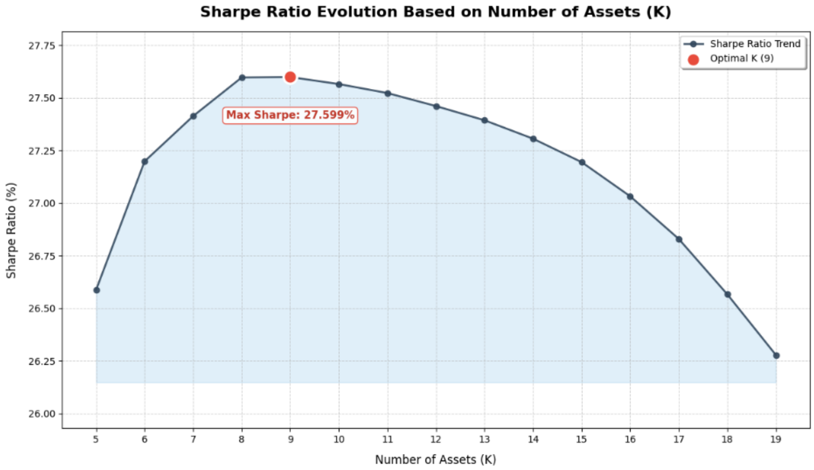

Figure 5. Sharpe Ratio Evolution of ABC

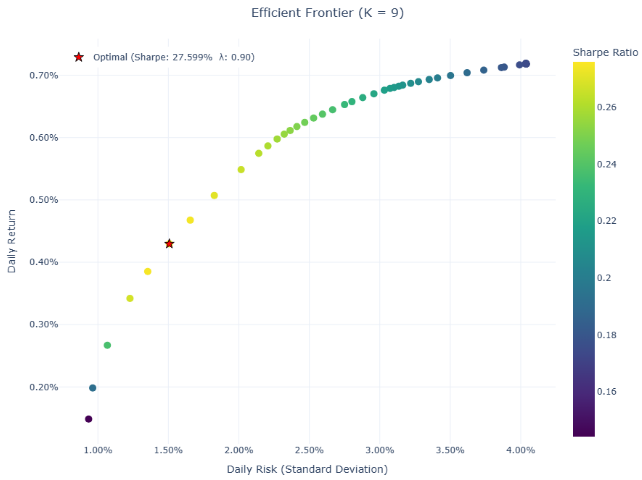

Figure 6. Efficient Frontier of ABC

Figure 6. illustrates the efficient frontier for the portfolio with K=9 by varying the risk-weighting parameter (λ) from 0 to 1. Each point on the curve depicts a specific risk-return combination, with the color gradient reflecting the Sharpe ratio levels. The optimal portfolio, identified by a red star, is achieved at λ=0.90, yielding the maximum Sharpe ratio of 27.599%. This signifies that the configuration provides the superior risk-adjusted return compared to all other tested parameter variations within the ABC algorithm. The specific weight allocation for each selected asset within this optimal portfolio is presented in Figure 7.

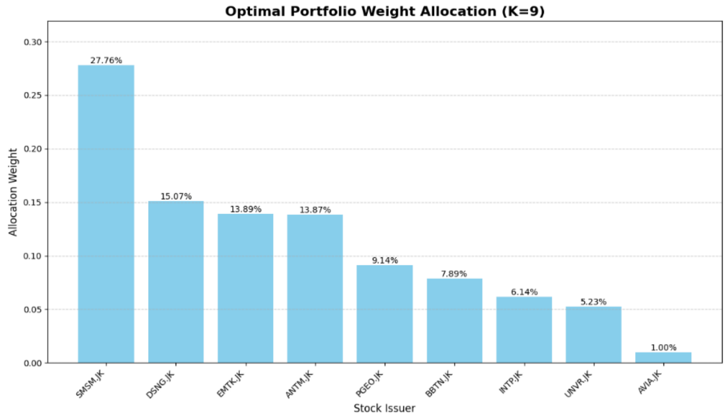

Figure 7. Portfolio Allocation using ABC

Figure 7 illustrates the optimal portfolio allocation generated by the ABC algorithm (K = 9), revealing a concentrated yet diversified structure in which a limited number of assets receive dominant weights. One stock clearly emerges as the primary contributor, accounting for approximately 27.76% of the total portfolio, indicating its strong risk–return profile within the optimization framework. The next tier of assets receives moderate allocations of 13% to 15%**,** suggesting a complementary role in enhancing diversification while maintaining return potential. In contrast, the remaining assets are assigned relatively small weights (below 10%) and serve as risk stabilizers, helping reduce overall portfolio volatility. The smallest allocation, close to 1%**,** reflects the algorithm's ability to include marginal assets primarily for diversification benefits rather than return maximization. This pattern highlights an important characteristic of cardinality-constrained optimization: while the number of selected assets is limited, the model still produces uneven weight distribution, prioritizing high-performing assets while retaining others for risk mitigation.

Determining the Best Algorithm

Following the portfolio optimization using PSO and ABC, the superior algorithm for constructing the optimal ESG portfolio under the CCMV model is determined. The selection is based on a comparative analysis of key performance indicators, return, risk, and Sharpe ratio as detailed in Table 2.

Table 2. Algorithm Performance Comparison

Algorithm	K	*Return*	*Risk*	*Sharpe*	*Running time*
PSO	8	0.433%	1.519%	27.594%	14 minutes
ABC	9	0.430%	1.507%	27.599%	9 minutes

The results indicate that both metaheuristic algorithms can generate well-diversified portfolios with relatively high Sharpe ratios under cardinality constraints. Specifically, the Artificial Bee Colony (ABC) algorithm produces a portfolio consisting of 9 assets with a Sharpe ratio of 27.599%, slightly outperforming Particle Swarm Optimization (PSO), which selects 8 assets and achieves a Sharpe ratio of 27.594%. Although the difference in Sharpe ratio is marginal, ABC demonstrates lower portfolio risk (1.507% vs. 1.519%) and significantly higher computational efficiency, requiring only 9 minutes compared to PSO's 14 minutes.

These findings are consistent with prior studies that highlight the effectiveness of metaheuristic algorithms in solving constrained portfolio optimization problems. Previous research has shown that both PSO and ABC can handle non-linear and combinatorial complexities; however, ABC is often reported to converge faster and provide more stable solutions in high-dimensional search spaces. The current results reinforce this evidence, suggesting that ABC may offer a more efficient balance between solution quality and computational cost.

From a broader perspective, the results suggest that incorporating cardinality constraints with metaheuristic optimization can produce practically implementable portfolios without significantly sacrificing performance (Cai et al., 2024). The small difference in returns, along with improved efficiency, implies that algorithm selection should consider not only accuracy but also computational practicality. For practitioners, ABC offers a reliable and time-efficient alternative for portfolio construction, while for researchers, the findings open opportunities to further explore hybrid or adaptive metaheuristic approaches to enhance robustness across varying market conditions.

CONCLUSION

This study demonstrates that the Artificial Bee Colony (ABC) algorithm outperforms Particle Swarm Optimization (PSO) in constructing an optimal portfolio within a cardinality-constrained mean–variance framework. ABC achieves a slightly higher Sharpe ratio (27.599%) while also offering greater computational efficiency, completing the optimization in a shorter execution time. These findings reinforce the capability of metaheuristic approaches to address complex, non-linear optimization problems involving cardinality constraints in portfolio selection. Despite these contributions, the study has several limitations. The analysis is restricted to a single index and a relatively short observation period of six months, which may limit the generalizability and robustness of the results across different market conditions. Future research is therefore recommended to expand the scope by incorporating multiple indices (Gönül & Omay, 2025), including sectoral and global market indices, and by extending the observation period to capture longer-term dynamics (Jin et al., 2025). Additionally, integrating more realistic constraints, such as transaction costs and explicit ESG scoring, into the optimization model could enhance practical relevance. Exploring hybrid metaheuristic techniques may also further improve optimization performance and portfolio stability.

ACKNOWLEDGMENTS

The authors gratefully acknowledge financial support from the Institut Teknologi Sepuluh Nopember (ITS) for this work, under the project scheme of the Publication Writing and IPR Incentive Program (PPHKI) 2026.

AUTHOR CONTRIBUTIONS

Conceptualization, JWI and NS; Methodology, JWI; Software, JWI; Validation, NS; Formal Analysis, JWI; Resources, JWI; Data Curation, JWI, MAN; Writing - Original Draft Preparation, JWI; Writing – Review & Editing, NS and MAN; Visualization, JWI; Supervision, NS and MAN; Funding Acquisition, Institut Teknologi Sepuluh Nopember (ITS).

CONFLICTS OF INTEREST

The author(s) declare no conflict of interest.

USE OF ARTIFICIAL INTELLIGENCE (AI)-ASSISTED TECHNOLOGY

The authors declare that no artificial intelligence (AI) tools were used in the generation, analysis, or writing of this manuscript. All aspects of the research, including data collection, interpretation, and manuscript preparation, were carried out entirely by the authors without the assistance of AI-based technologies.

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