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Constructing the Concept of Geometric Transformation through APOS Theory: A Perspective on Learning Readiness

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Abstract

Concept construction is essential for developing mathematical abilities such as conceptual understanding and problem-solving. The APOS theory (Action, Process, Object, Scheme) offers a framework for conceptual development through mental activity sequences. A key factor influencing concept construction is students' learning readiness, which is strongly tied to their initial abilities. This study investigates the relationship between learning readiness and concept construction, as well as describes students' concept construction profiles based on APOS theory. Employing a mixed-method approach with an explanatory sequential design, the study involved 34 eleventh-grade students from a public high school in Gresik. Data collection techniques included cognitive diagnostic assessments, written tests, think-aloud protocols, interviews, and classroom observations. Quantitative data were analyzed using regression and correlation analyses, while qualitative data were processed through reduction, display, and conclusion drawing. Findings revealed a very strong, positive correlation (r=0.9046) between learning readiness and concept construction, suggesting that higher readiness aligns with more complex conceptual structures. Additionally, three profiles emerged: students with advanced readiness displayed complete analytical constructions; those developing readiness showed pseudo-complete constructions; and newly developing students exhibited incomplete geometric constructions. Future studies should explore APOS-based instructional models tailored to readiness levels and validate findings across broader mathematical topics and grade levels.

Kata Kunci: APOS theory; concept construction; geometric transformation; learning readiness.

INTRODUCTION

Concept construction is a series of activities to reorganize previously constructed mathematical concepts through new mathematical structures. A person's way of thinking will influence how he or she constructs a concept (Scheiner & Pinto, 2014) (Imamuddin et al., 2019). According to (Anggraini et al., 2018) the concept construction process is an activity of constructing a concept by connecting it to other concepts. The process of developing a concept involves steps taken by an individual to create a new idea or understanding (Rosyidi & Hasanah, 2022). Mathematics learning requires emphasis on the student's learning process, where they construct their knowledge through experience and according to their stage of cognitive development (Mudrikah, 2015). This is in line with the emphasis on learning in the Independent Curriculum, namely that students actively construct their knowledge. Knowledge will be formed if students carry out the construction process actively, so that previous concepts are used to learn the next concept (Mayorova et al., 2021). Guidance from teachers or peer tutors and learning strategies will be very helpful in constructing a mathematical concept (Pramesti & Prasetya, 2021) (Ulum & Pujiastuti, 2020).

The development of concepts is essential for supporting other mathematical skills, including understanding concepts and solving problems (Ismail et al., 2021). In line with this, APOS theory can be used in building mathematical concepts through mental activities, namely actions, processes, objects, and schemes (Ed Dubinsky, 2017). APOS theory is widely recognized in the field of mathematics as a useful framework for assessing and enhancing students' comprehension and learning of mathematical ideas (Fuentealba et al., 2017) (López et al., 2016). APOS theory serves as a theoretical framework in Mathematics Education that describes, from a cognitive perspective, how students construct or grasp mathematical concepts by building on their prior knowledge, which gradually evolves into new understanding (Martinez & Parraguez, 2017). According to APOS theory, mathematics instruction should support students in utilizing their existing mental frameworks while also guiding them to construct more robust structures to address increasingly complex mathematical problems (Arnawa et al., 2019). APOS Theory is a modification of Piaget's concepts, applied to understanding how individuals acquire mathematical knowledge through a sequence of stages: Actions, Processes, Objects, and Schemas. (Nagle et al., 2019). APOS is a theory that focuses on students' mental attitudes during learning in building mathematical concepts. Mathematical concepts are viewed as outcomes of building and rebuilding mathematical objects. These processes involve repeated mathematical actions that occur in a structured sequence, eventually forming objects within a schema used to address mathematical problems (Díaz-Berrios & Martínez-Planell, 2022). In APOS theory, mental construction refers to the transformation of internalized actions into mental processes, which are subsequently encapsulated as mathematical objects. These objects can later be broken down again into their underlying processes.

In the APOS theory, it is seen that the construction of individual understanding does not always run straight between one stage and another. Between these stages can move back and forth according to the situation required. Then from the action, process, and object stages are organized into a scheme. (Arnon, I., Cottrill et al., 2014) describes the mental structure and mechanism of understanding construction based on the APOS theory which can be seen in Figure 1 below.

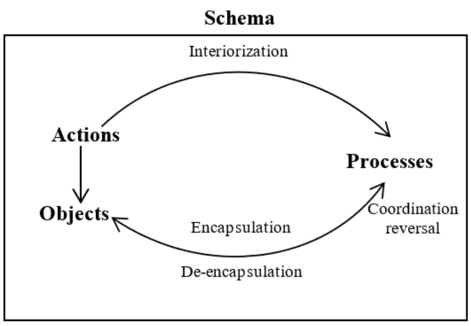


Figure 1. Mental Structure and Mechanism of Understanding Construction Based on APOS Theory

At the action stage, students only solve problems procedurally. If the action activity is carried out repeatedly, then students will be able to carry out the transformation steps without real assistance. If students have been able to do this, then students have reached the process level. The object level can be achieved if students have been able to understand the concept of a material. At the schema level, students have been able to associate certain concepts with other similar concepts that aim to solve problems (Simanjuntak et al., 2022).

Previous research found that students have difficulty in understanding mathematical concepts and cannot recognize their various representations, indicating low concept construction abilities (Uegatani et al., 2024). Other studies have revealed that students have difficulty in transforming between mathematical and graphical representations, indicating challenges in the construction of mathematical concepts (Lin et al., 2013). Previous studies have also found that students have significant difficulties in understanding basic mathematical concepts, which results in their low ability to build more complex mathematical concepts. These difficulties are caused by ineffective learning approaches and weak initial conceptual foundations (Lima et al., 2019). Other studies have also shown that low-achieving students have great difficulty in constructing mathematical concepts, especially when having to use various representations and manipulative aids. This results in a less comprehensive understanding of the concept (Hodgen et al., 2024). In addition to general mathematical concepts, other studies have also found that students' difficulties in constructing geometric concepts affect their understanding of proof and the proof process. Students have difficulty in identifying common elements of two shapes and using self-attributes of a single figure, which affects their ability to prove theorems (Haj-Yahya et al., 2016).

Geometry is an essential component of mathematics education that students need to understand well (Turmuzi et al., 2021) (Anwar & Rofiki, 2018). Geometric transformation is a topic that requires in-depth conceptual construction. Geometry encompasses various intricate concepts, covering mathematical studies related to shapes, sizes, the relative positioning of lines and planes, as well as the characteristics of objects in both two- and three-dimensional spaces. This encompasses geometric transformations like reflection, dilation, and contraction within both planar and (Fauziyah & Husniati, 2023) spatial settings, as well as coordinate representation of elements such as lines, planes, and conic shapes (Harel, 2019). At the high school level, geometric transformations are presented in matrix form. On the other hand, geometric transformation concepts such as translation, reflection, rotation, and dilation must be mastered as a prerequisite in studying the material on geometric transformation composition.

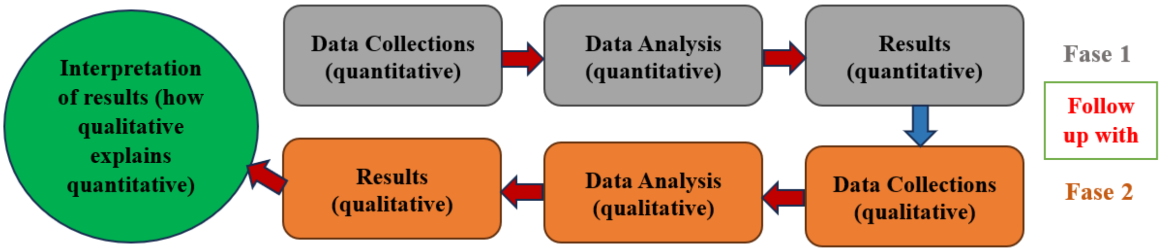
One important aspect of a student's learning needs in differentiated learning is learning readiness. In differentiated learning, learning readiness is related to students' initial abilities. Initial abilities are basic skills that students must have before starting new learning, reflecting their readiness to receive material presented by the teacher (Mulyono et al., 2018). So that students' learning readiness can be detected through initial assessments or diagnostics containing questions about prerequisite material (Gustavo et al., 2019). The learning readiness of one student to another is certainly different because it is greatly influenced by the student's learning experience. The difference in readiness affects the level of understanding of the material given. It can be concluded that students with different learning readiness will have different concept construction processes in learning a material.

Several studies related to concept construction based on APOS theory have been conducted. In the study of Rosyidi et al. (Rosyidi & Hasanah, 2022) regarding the process of constructing new concepts by students based on gender was examined. Other studies have also been conducted by Suriyah et al., concept construction based on APOS theory is reviewed from differences in levels of creative thinking ability (Suriyah et al., 2022). Another study that uses APOS Theory as the main theoretical framework is a study conducted by Martínez et al. APOS Theory is used to analyze and explain how students develop geometric understanding of partial derivatives through the stages of action, process, object, and schema. Researchers identify how students mentally construct concepts and connect geometric representations with abstract mathematical concepts. The subjects of the study were reviewed from different levels of geometric understanding (Borji et al., 2024). Other studies discuss the application of APOS Theory in mathematics teaching, including the construction of geometric transformation concepts. The study only wants to show the importance of a constructivist approach in understanding mathematical concepts in depth. This study does not explore the differences in students in building understanding of mathematical concepts through the four mental stages in APOS theory that are interrelated in any way (Oktaç et al., 2019). Other research developed a teaching model for mathematical concepts using APOS Theory. The results showed that this approach helped students build a deeper understanding of the concept through the stages of action, process, object, and schema (Salgado & Trigueros, 2015). While research that reveals the profile of students' mathematical concept construction using the APOS theoretical framework, reviewed from the differences in students' learning readiness levels, has not been conducted. Thus, the researcher claims that this study has a different novelty from previous studies. The learning readiness referred to in this study is not general learning readiness. Learning readiness in this study is more specific to mastery in understanding the prerequisite material of material to be studied. This seems to be a gap that previous studies have defined as learning readiness, as the psychological or mental readiness of students. Readiness in understanding the prerequisite material is chosen as the basis for determining the subject because of the hierarchical characteristics of mathematics, if one material cannot be understood well, then the next relevant material will also be difficult to understand. This is what happens in real classes, students will find it difficult to construct certain concepts, because they have not mastered the prerequisite material for the concept.

Based on the background that has been described, this study seeks to answer two main research questions: (1) What is the relationship between students' learning readiness and the construction of mathematical concepts? and (2) how is the profile of students' construction of geometric transformation concepts based on APOS theory characterized in terms of their learning readiness?

METHOD

This type of research is a mixed-methods research with an explanatory sequential design. Explanatory sequential design is the use of two studies in stages, starting from quantitative research and continuing with qualitative research (Creswell & Creswell, 2018). The research flow of the explanatory sequential design type can be seen in Figure 2.



(Adapted from (Creswell & Creswell, 2018))

Figure 2. Explanatory Sequential Design Flow

The quantitative research used is a correlational research type, while the qualitative research used is a descriptive research type. The findings in this study are presented in the form of statistical and narrative analysis. Researchers, cognitive diagnostic assessment test questions, concept construction test questions, and semi-structured observation sheets are the instruments used in this study. This research was conducted at one of the State Senior High Schools in Gresik Regency, East Java, from November to December 2024. Participants in this study were 34 students in grades XI-8 who were studying geometric transformation material. This class was selected using **purposive sampling**, considering that they were currently studying **geometric transformation**, which is the focus of the research. The class was also chosen because it was deemed representative for exploring the process of mathematical concept construction based on APOS Theory, due to the diverse range of students' **learning readiness**. Through purposive sampling, the researcher was able to purposefully examine the relationship between students' learning readiness and the stages of concept construction they developed. The participants were given a cognitive diagnostic assessment to map their learning readiness. The cognitive diagnostic assessment used refers to the independent curriculum diagnostic assessment.

The results of the assessment were processed and categorized into three groups as shown in Table 1 below.

Table 1. Cognitive Diagnostic Assessment Score Categories Based on Learning Readiness

No	Learning Readiness	Cognitive Diagnostic Assessment Score Intervals
1.	New Learning Readiness Develops	$x < 60$
2.	Readiness to Learn is Developing	$60 \leq x < 80$
3.	Readiness to Learn Has Become Advanced	$80 \leq x$

Description: x is the cognitive diagnostic assessment score

Based on Table 1 above, the research participants were grouped into 3 groups based on the results of the diagnostic assessment of learning readiness, namely students with the learning readiness category group "developing" ($x < 60$), learning readiness "newly developing" ($60 \leq x < 80$), and learning readiness "advanced" ($x \geq 80$) with a score range of 0 to 100.

Then, the learning readiness group was obtained. Then students were given test questions that had to be answered on the sheet provided while saying out loud what they were thinking (think-alouds). The results of students' think-alouds were stored in the recordings provided. The question instrument used consisted of 1 descriptive question related to how to construct an image due to the composition of two consecutive reflections on parallel lines. After the test scores were processed, one student with the highest score was selected from each group to obtain a description of the concept construction profile by considering their communication skills. This technique is called the purposive sampling technique. Purposive sampling is a technique for selecting research samples by considering something. Then, continued with semi-structured interviews and observations were conducted to complete the data. The indicators used to reveal the concept construction process in this study are those that can be seen more clearly in Table 2 below.

Table 2. Concept Construction Indicators Based on APOS Theory

Mental Structure	Mental Mechanism	Indicator
Action	Interiorization	Identifying information or concepts contained in the problem.
Process	Coordination	Analogizing the problem of the composition of geometric transformations with the composition of functions. Determining the second image, which is the second reflection of the image resulting from the first reflection.
	Reversal	Tracing back the general form of the matrix equation
Object	Encapsulation	Determines the image result by the composition of successive reflection transformations on two lines parallel to the Y axis.
	De-encapsulation	Can decapsulate an object back into the process from which it originated or decompose a thematized scheme into its various components.
Scheme	Thematization	Relate the image result by the composition of successive reflection transformations to two lines parallel to the Y axis as a translation of the initial abscissa by twice the distance between the reflection lines.

To ensure data credibility, validation was performed by cross-checking information from the same source using various methods. In this research, triangulation was applied through the use of tests, interviews, observations, and the think-aloud method. The study employed both quantitative and qualitative data analysis approaches. Quantitative analysis involved the use of regression analysis and correlation coefficients, while qualitative data were analyzed using the Miles and Huberman model, which consists of data reduction, data display, and conclusion drawing.

RESULTS

Participants in this study were given an initial assessment/cognitive diagnostic to obtain learning readiness category groups. The results of the cognitive diagnostic assessment of learning readiness can be seen in Table 3 below.

Table 3. Cognitive Diagnostic Assessment Result Data

No	Readiness to Learn	Many students	Percentage (%)
1.	New Learning Readiness Develops	10	29,41
2.	Readiness to Learn is Developing	18	52,94
3.	Readiness to Learn Has Become Advanced	6	17,65

Based on Table 3 above, 3 groups of learning readiness were obtained from 34 students with the following details. 10 students with learning readiness were just developing (29.41%), 18 students with learning readiness were developing (52.94%), and 6 students with learning readiness had advanced (17.65%).

Prerequisite Test

Before conducting a hypothesis test, a prerequisite test must be conducted, including a normality test and a linearity test to determine whether the hypothesis test conducted is parametric or non-parametric.

Normality Test

Table 4. Summary of Data Normality Test

Variabel	Statistic	Df	Sig
Learning Readiness	0,110	34	0,150
Concept Construction	0,112	34	0,150

Source: Minitab Data Processing Results, 2024

From the results of the Kolmogorov-Smirnov normality test, in Table 4, the significance value for learning readiness and concept construction is 0.150, which means it is greater than the probability value of 0.05. From the results of the normality test, it can be concluded that learning readiness and concept construction are normally distributed.

Linearity Test

Table 5. Linearity Test

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	1	3696.2	3696.16	303.37	0.000
Readiness to Learn	1	3696.2	3696.16	303.37	0.000
Error	32	389.9	12.18		
Lack-of-Fit	21	166.0	7.91	0.39	0.970
Pure Error	11	223.8	20.35		
Total	33	4086.0			

Source: Minitab Data Processing Results, 2024

Based on Table 5, it shows that the significance value is 0.000, which means it is smaller than the probability value of 0.05, so it can be concluded that the relationship between learning readiness and concept construction is linear.

Hypothesis Testing

Based on the prerequisite test results, the data were found to be normally distributed and linear, thus enabling further analysis using parametric correlation tests.

Correlation Analysis

Table 6. Pearson Correlation Analysis and Statistical Model

Pairmise Pearson Correlations					Model Summary			
Sample 1	Sample 2	Correlation	95% CI for p	P-Value	S	R-sq	R-sq(adj)	R-sq(pred)
Concept Construction	Readiness to Learn	0.951	(0.904, 0.976)	0.000	3.49049	90.46%	90.16%	88.69%

Source: Minitab Data Processing Results, 2024

Because the P-value is $0{,}000$ ($< 0{,}05$), It can be concluded that there is a relationship between learning readiness and concept construction. The correlation value obtained is positive at 0.951, which means that learning readiness and concept construction have a very strong and positive relationship, namely, the higher the student's learning readiness, the more complex the constructed concept becomes. The coefficient of determination (R^2) for learning readiness influences concept construction by 90.46% and as much as 9.54% is influenced by other factors. The distribution of learning readiness and concept construction data can be seen in Figure 3.

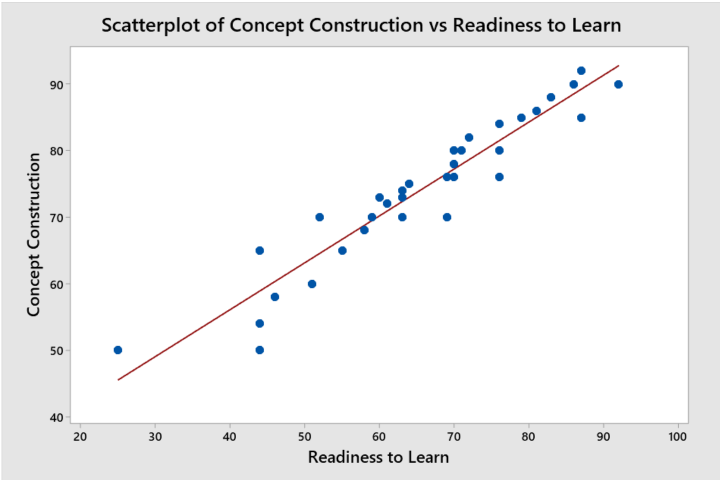


Figure 3. Distribution of Learning Readiness and Concept Construction Data

This scatterplot graph is used to display the relationship between two quantitative variables: readiness to learn and concept construction ability in mathematics. The graph clearly shows a strong positive correlation between these variables. In other words, the higher a student’s readiness to learn, the greater their ability to effectively construct mathematical concepts.

The linear regression line shown represents the average trend of the relationship between readiness to learn and concept construction. The data points clustered closely around this regression line indicate that the relationship between the two variables is consistent and stable across the data range.

Additionally, the data points are tightly grouped around the regression line, which signifies a very strong correlation between readiness to learn and concept construction ability. There are almost no outliers, suggesting that the data is fairly homogeneous and supports the validity of the statistical conclusions.

Visually, this pattern suggests a Pearson correlation coefficient typically above 0.8. Referring to the previous research context where a correlation of 0.90 was reported, this graph strongly supports those findings. The linear and tight distribution of data reinforces the presence of a significant positive relationship between students’ readiness to learn and their ability to construct mathematical concepts.

Therefore, this scatterplot clearly illustrates that the more ready a student is to learn, the more complex and developed the mathematical concepts they can build. This provides strong visual support for the claim that readiness to learn is an important factor in students’ conceptual understanding of mathematics.

Profile of Student Concept Construction

Then, from the results of the learning readiness grouping, one student was selected using a purposive sampling technique to explore more deeply about the profile of their concept construction.

Subjects with advanced learning readiness were coded with S1, subjects with developing learning readiness were coded with S2, and subjects with newly developing learning readiness were coded with S3, and the following profile was obtained.

Student Concept Construction Profile with "Advanced" Learning Readiness

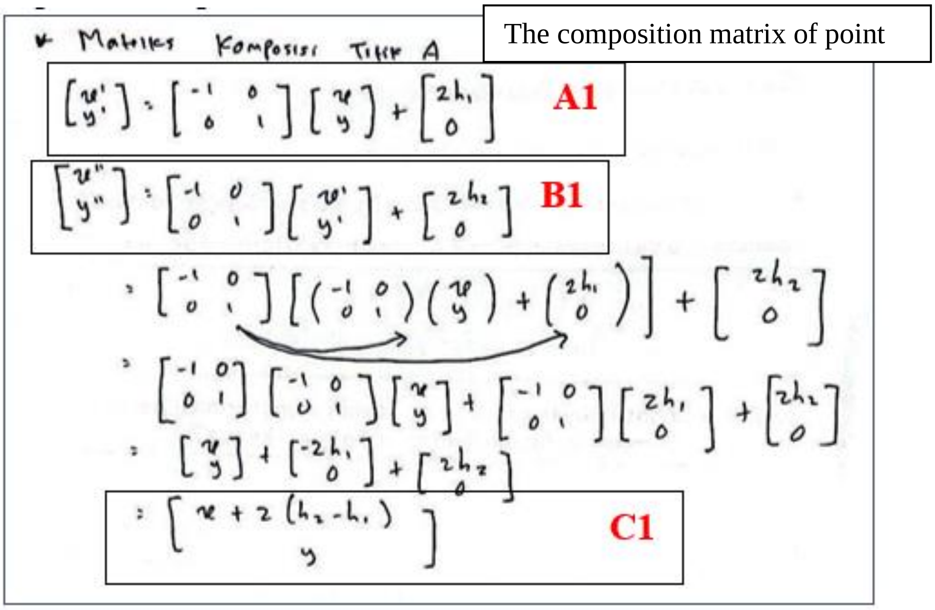


Figure 4. S1 Answer Results

In Figure 4, parts A1, B1, and C1, respectively, show the results of S1's written interpretation of the reversal, coordination, and encapsulation mechanisms.

To solve the problem, S1 started reading the problem carefully.

S1: "Determine the image of point A(1,1) reflected on the line x = 2 followed by reflection on the line x = 5."

This shows that S1 is trying to understand the problem. Furthermore, S1 **internalizes** by identifying information or concepts contained in the problem to be processed in his mind. The information identified is $A(x, y)$ is the starting point reflected on the line $x = h_1$ which produces the image $A'(x', y')$ then, continued reflection on the line $x = h_2$ produces the image $A''(x'', y'')$. His is based on the following transcript of S1's think-alouds.

S1: "So, if we have a point, let's say $A(x, y)$reflected on the line x, let's say $x = h_1$, so the reflection is $A'(x', y')$ then, if we continue the reflection, let's say $x = h_2$, then the result is $A''(x'', y'')$."

S1 uses the symbols h_1 and h_2 to distinguish constants in the equation of a line parallel to the Y-axis, and he chooses the symbol h because in the knowledge he has previously obtained, the equation of a line parallel to the Y-axis has the form x = h. This is known from the following interview transcript.

P: "Why do you use the symbols $h_1$and h_2?"

S1: "Oh, that's to differentiate between the first and second lines. Because I learned in previous materials that vertical lines are written as x = h."

In the mental structure of action, it can be seen that S1 has a strong basic understanding, making it easier to internalize.

Then, **coordination** occurs when S1 analogizes the problem as in the composition function. Consider the following think-aloud excerpt.

S1: "By using a formula like the composition function, namely T_2 circle T_1. In this formula, T_1 is still the first to be worked on, and then it is continued $T_{2.}$ T_1How's the first reflection and T_2shows the second reflection.

From the think-aloud transcript above, S1 identifies that the concept of composition in geometric transformation is the same as the concept that has been studied previously, namely the composition of functions. So S1 writes the composition of geometric transformation as $T_2 \circ T_1$

Reversal occurs when S1 retraces the general form of the reflection matrix equation to the line $x = h_1$,namely $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 2h_1 \\ 0 \end{bmatrix}$. This can be seen in Figure 4 section A1. Furthermore, **coordination** occurs again when S1 determines the second image, so the image result is the second reflection of the image result in the first reflection. In Figure 4 part B1, S1 wrote it as $\begin{bmatrix} x'' \\ y'' \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x' \\ y' \end{bmatrix} + \begin{bmatrix} 2h_2 \\ 0 \end{bmatrix}$.

Encapsulation occurs S1 determines the image result by the composition of successive reflection transformations on two lines parallel to the Y axis is $\begin{bmatrix} x'' \\ y'' \end{bmatrix} = \begin{bmatrix} x + 2(h_2 - h_1) \\ y \end{bmatrix}$. Look at Figure 4 part C1. However, in S1 no de-encapsulation was found.

Thematization occurs when S1 shows a connection between the image result by the composition of successive reflection transformations on two lines parallel to the Y axis and an abscissa as a translation of the initial abscissa with twice the distance between the reflection lines. This is further explored in the following interview.

P: "From where can you conclude that the image result is the sum of the initial abscissa with twice the distance between the reflection lines?"

S1: "If the addition is the same as translation, then I remember that the distance is the second minus the first, so h2 minus h1 is the distance."

This study illustrates how S1's mental structure is formed and developed in solving the problem of successive reflections on two lines parallel to the Y-axis. Mental structure refers to the cognitive framework consisting of action, process, object, and schema, which interact dynamically in mathematical understanding. At the Action stage of the mental structure, S1 performs direct actions such as carefully reading the problem and applying the reflection procedure step-by-step. For example, S1 explicitly thinks about how point A is reflected on the line $x = 2$, then the result is reflected again on the line $x = 5$. This shows the initial mentalization as concrete actions applied to the problem information.

At the Process stage, S1 begins to build a more abstract mental process by understanding reflection not as separate steps but as a composition of functions. S1 can view the reflection as a sequential transformation process where the result of the first reflection becomes the input for the second reflection. This indicates that S1's mental structure has progressed from action to internalizing a continuous process. The Object stage in the mental structure is evident when S1 successfully encapsulates the process of composition of reflections into a single mathematical entity or object that can be understood and manipulated holistically. S1 concludes that two successive reflections over vertical lines can be represented as a translation on the x-coordinate of the original point. Thus, S1 no longer sees the reflection steps separately but as one whole transformation object with specific properties.

The Schema emerges when S1 organizes different concepts into an integrated framework that connects reflection, function composition, translation, and the distance between lines systematically. S1 can explain the mathematical relationships underlying the successive reflections and provide consistent reasoning based on prior knowledge about vertical lines and reflections. This demonstrates a mature mental structure where various concepts are integrated into a well-structured way of thinking.

However, the study also shows that de-encapsulation is not observed in S1's mental structure, meaning S1 has not yet demonstrated the ability to explicitly break down the encapsulated object back into the underlying processes. This suggests that although S1 has mastered a complex schema, the ability to deconstruct that schema into component processes is not yet apparent in his reasoning. Overall, S1's mental structure in solving the reflection problem develops progressively from concrete action, ongoing mental process, recognition of the transformation object, to an integrated conceptual schema. This illustrates how mathematical understanding can develop gradually and systematically in line with the APOS theory and mental structure framework.

Student Concept Construction Profile with "Developing" Learning Readiness

From S2's answers and think-alouds, S2 solved the composition problem of two consecutive reflections on two parallel lines Y by writing them in a matrix equation. S2 worked on the transformations one by one which can be seen on the answer sheet in Figure 5 below.

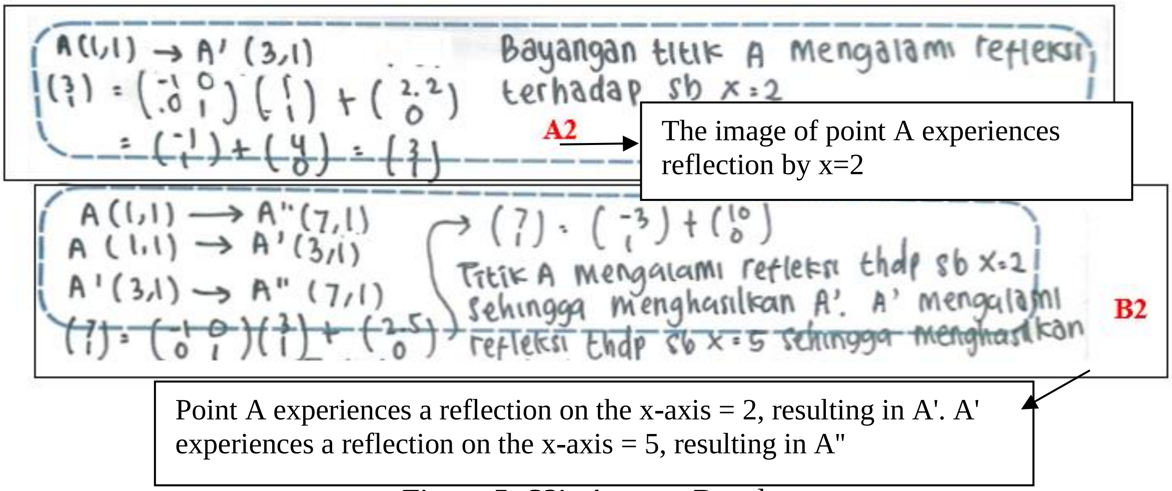


Figure 5. S2's Answer Results

In Figure 5, it can be seen that S2 finds the second image of point A using the matrix approach. S2 does it step by step by finding the first image and then using the first image as the starting point to find the second image.

S2 **internalizes** by identifying information or concepts contained in the problem to be processed in his mind. The information identified is a point $A(1,1)$ as the starting point which is reflected on the line $x = 2$ then continued with reflection on the line $x = 5$. This is known from the results of S2's think-aloud aloud which says

S2: "Given point $A(1,1)$ which will be reflected on the line $x = 2$ then it continued its reflection on the line $x = 5$. How many image points are there"

Then an implicit **reversal** occurs when S2 retraces the general form of the reflection matrix equation on the line parallel to the Y axis because S2 immediately coordinates by determining the image A'. This can be seen in Figure 5, part A2.

Then, there is **re-coordination** to determine the second image, then S2 performs a transformation again with the first image as the starting point. This can be seen from the following interview transcript.

P: "What do you do after getting the first image result?"

S2: "Because there is another transformation, the method is the same as before. But the starting point is not A but A'. So from the first image, we use it as the starting point."

For S2's answer regarding the second image from point A or A'can be seen in Figure 5 section B2. **Encapsulation** occurs S1 determines the image result by the composition of sequential reflection transformations on two lines parallel to the Y axis is $A'\begin{bmatrix} x'' \\ y'' \end{bmatrix} = \begin{bmatrix} 7 \\ 1 \end{bmatrix}$. Similar to S1, S2 does not show a mental de-encapsulation mechanism.

Thematization occurs when S1 shows a connection between the image result by the composition of sequential reflection transformations on two lines parallel to the Y axis as a result of translation along the perpendicular direction of the reflection lines and its abscissa will increase. This is proven by the following think-aloud excerpt.

S2: "So the conclusion is, the composition of two reflections on two parallel lines produces a translation along the perpendicular direction of the lines, and the matrix will be added with the second reflection."

Subject S2 demonstrates a fairly complex mental construction process in solving the problem of composing two consecutive reflections on two parallel lines, which can be analyzed through the mental mechanisms in APOS theory.

Interiorization occurs when S2 begins to recognize and internalize important information in the problem, namely the initial point to be reflected and the two parallel lines as axes of reflection. This is evident from S2's explicit thinking about the point that will first be reflected on the first line, and then the result will be reflected on the second line. Thus, S2 processes this information mentally as the foundational material for solving the problem.

Next, S2 performs Coordination, which is organizing and coordinating various pieces of information and concepts simultaneously in the mind. For example, S2 integrates the position of the initial point, the first reflection result, and the second reflection line to proceed with the next transformation. S2 does not simply view reflection as a single action but can unite the result of the previous action as a new input for the next one. This shows that S2 can coordinate multiple transformation objects simultaneously.

Then, Reversal occurs, which is S2's ability to reverse or retrace the transformation process previously performed. For instance, S2 can recall and retrace the general form of the reflection matrix on lines parallel to the Y-axis and then apply it again in the next transformation context. This reversal helps S2 evaluate and ensure the correctness of the steps taken in solving the problem.

At the Encapsulation stage, S2 integrates the composition of two consecutive reflections into a single mental object, namely a transformation producing a translation along the direction perpendicular to the reflection lines. S2 perceives this reflection composition no longer as two separate actions but as a new single transformation that can be recognized and applied as a whole. This indicates the formation of a more complex object concept in S2's mind.

However, regarding De-encapsulation, which is the ability to unpack a mental object back into simpler action components, S2 does not show clear signs. That is, S2 tends to understand the reflection composition as a unity without breaking it back down into more basic action steps. This indicates a limitation in mental flexibility in manipulating this concept.

Finally, through Thematization, S2 connects the result of the reflection composition with a broader concept, namely translation. S2 realizes that the composition of two reflections on parallel lines results in a translation along the direction perpendicular to those lines, and that the transformation matrix of the composition is the sum of the two reflection matrices. Thus, S2 develops an integrated and comprehensive schema linking various transformation concepts into a more general framework.

Overall, S2's mental construction can be described as a journey from recognizing and processing initial information (interiorization), organizing concepts (coordination), retracing processes (reversal), forming the transformation object (encapsulation), to developing a broader conceptual schema (thematization). However, the limitation in the de-encapsulation mechanism suggests that S2 still needs to develop the ability to flexibly unpack and reconstruct the transformation concepts into more basic forms.

Profile of Student Concept Construction with "Newly Developing" Learning Readiness

The results of S3's answer can be seen in Figure 6 below.

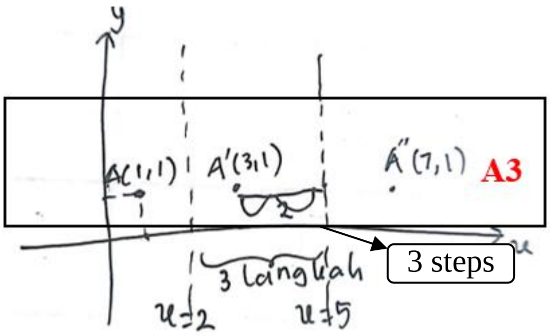


Figure 6. S3's Answer Results

Figure 6 shows that S3 uses the Cartesian coordinate approach in determining the image of point A by transforming the line x=2 followed by transforming the line x=5.

S3 **internalizes** by identifying information or concepts contained in the problem to be processed in his mind. The information identified is a point $A(1,1)$The starting point that is reflected on the line x=2 then continued with reflection on the line x=5. This is known from the results of S3's think-aloud aloud which states

S3: "There is a point $A(1,1)$ that will be reflected on the line x =2 then reflected again on the line x=5. Determine the image point"

Then the reversal in S3 occurs by tracing the image of the reflection geometrically. This is because S3 forgot the general form of the reflection matrix equation. This is known from the following interview transcript.

P: "What steps did you take to solve the problem?"

S3: "So I just drew it, because I forgot the image formula for the reflection matrix"

P: "Then how can you write the image $A'(3,1)$? Where did you get it from?"

S3: "If the mirror means the distance is the same, well, because the x point is 1, the mirror x = 2 means it jumps one, so it jumps one too, the image becomes at x = 3 and the y remains the same. What changes is x" From the interview above, it can also be seen that S3 also **coordinates** in determining the image, the distance of the starting point to the mirror is the same as the distance of the image point to the mirror.

Then, there is **re-coordination** to determine the second image, then S3 performs a transformation again with the first image as the starting point. This can be seen from the following interview transcript.

P: "What did you do after getting the first image result?"

S3: "So the method is the same, the distance from the point to the mirror is the same as the distance from the image point to the mirror. If it is described as 2 steps, it means the second image is at (7,1)"

P: "Then, what do you mean by writing 3 steps in your answer"

S3: "Oh no, that's wrong"

For S3's answer regarding the second image from point A or A' can be seen in Figure 6 section A3.

Encapsulation occurs S1 determines the image result by the composition of sequential reflection transformations to two lines parallel to the Y axis is $A''(7,1)$Similar to S1 and S2, S3 does not show a mental de-encapsulation mechanism.

Thematization does not occur because S3 has not reached the stage of linking the image results by the composition of sequential reflection transformations to two lines parallel to the Y axis as a translation of the initial abscissa with twice the distance between the reflection lines. S3 only understands that if the reflection is on a line parallel to the Y axis, the abscissa x changes. This is proven by the following think-aloud excerpt.

S3: "So the conclusion is, what changes is x, y does not change"

Subject S3 demonstrated a basic conceptual understanding in solving the problem of the composition of two reflections over parallel lines. S3's thought process can be analyzed through the stages of **APOS theory**, which include the mental mechanisms of **interiorization, coordination, reversal, encapsulation, de-encapsulation**, and **thematization**.

At interiorization, S3 began solving the problem by mentally identifying the key elements. He understood that the initial point would be reflected over the line x=2, and the result would then be reflected again over the line x=5. The think-aloud statement: *"There is a point that will be reflected on the line x = 2 and then again on the line x = 5…"* shows that S3 was capable of **interiorization**, meaning he could process and internalize the given information mentally before taking action.

S3 then proceeded with **coordination**, where he related the position of the point to the mirror line and determined its image based on the concept of equal distance. Although S3 did not use algebraic forms or reflection matrices, he understood that the distance from the original point to the mirror is the same as from the mirror to the image. For example, when the point was one unit to the left of the line x=2x = 2x=2, he knew the image would be one unit to the right at x=3x = 3x=3. Coordination occurred again when S3 used the result of the first reflection as the starting point for the second reflection. This indicates that S3 could structure a sequence of transformations with consistent spatial reasoning.

S3 also demonstrated **reversal**, though not through symbolic or algebraic reasoning. Instead, he used geometric drawing and visualization. In the interview, S3 admitted he could not recall the reflection matrix formula, so he relied on visual intuition to retrace the reflection steps. This shows that reversal occurred conceptually and visually, rather than symbolically.

After performing two reflections in sequence, S3 viewed the final image as a result of one complete transformation. He no longer explicitly separated the steps but regarded the sequential reflections as one integrated process. This reflects **encapsulation**, in which multiple actions are mentally grouped and treated as a single object of thought.

Although encapsulation occurred, there was no evidence that S3 was able to perform **de-encapsulation**—that is, to break down the composed transformation into its individual reflection components for further analysis. Once the final result was obtained, S3 did not show cognitive flexibility to deconstruct it. This indicates a limitation in the recursive manipulation of mathematical objects.

Unlike other subjects, **thematization did not emerge** in S3's reasoning. He did not reach the stage of generalizing the result of two reflections as a translation. While S3 understood that reflecting over vertical lines changes the x-coordinate, he did not connect this observation to a broader concept such as translation. This is evident in his think-aloud statement: *"So the conclusion is, what changes is x, y stays the same."* This suggests S3's understanding remained local and procedural, without reaching higher levels of abstraction.

Overall, subject S3 demonstrates development at the **action** and early **process** stages of APOS theory. He can identify relevant information, coordinate two reflections visually, and logically organize the sequence of transformations. However, he does not exhibit **de-encapsulation**, and **thematization** has not yet been achieved, as he is unable to generalize the composition of reflections as a type of transformation such as translation.

Therefore, S3's mental construction still relies heavily on spatial and visual reasoning, without formal symbolization or conceptual generalization. This indicates that S3's mathematical schema development remains at an early to intermediate stage and needs further support to evolve into abstract and generalized mathematical thinking.

What happened in S3 is in line with (Noviyla et al., 2023) findings which state that students' conceptual reconstruction abilities in terms of APOS theory are inadequate because they are not used to developing their knowledge and lack understanding of prerequisite materials. Normally, at the cognitive stage, S3 students can construct transformation compositions using matrices analytically.

DISCUSSION

From the results of the study above, the differences in indicators in constructing concepts are presented in the following Table 7.

Table 7. Indicators that emerge from the subject

Mental Structure	Mental Mechanism	S1	S2	S3
Action	Interiorization	√	√	√
Process	Coordination	√	√	√
	Reversal	√	√	√
Object	Encapsulation	√	√	√
	De-encapsulation	-	-	-
Scheme	Thematization	√	√	-

The table reveals three distinct types of concept construction. Students categorized as having 'advanced' learning readiness demonstrate a relatively comprehensive analytical construction. This is considered comprehensive because, in solving problems, these students engage in processes such as interiorization, coordination, reversal, encapsulation, and thematization. However, the de-encapsulation process was not observed. Their problem-solving approach is deemed analytical due to their reliance on algebraic reasoning. These are the results of the study (Rofiki et al., 2020) that subjects with high mathematical abilities can meet the APOS indicators well, namely students can solve cube and cuboid problems correctly at the action stage. During the process and object stages, participants demonstrate the ability to explain in detail the procedures for calculating surface area and volume by comparing two differently sized geometric shapes. At the schema stage, they develop a structured and coherent understanding of the concepts related to the surface area and volume of cubes and cuboids.

Meanwhile, students with "developing" learning readiness are said to have a type of analytical construction that tends to be pseudo-complete because it is the same as students with "advanced" learning readiness using algebraic solutions, but all because an implicit reversal mechanism is found. Compared to the results of the study (Rofiki et al., 2020) that subjects with moderate mathematical abilities were able to explain mental construction at the action and process stages well, even though there were errors at the process stage. In the next stage, the subject compared two cube and cuboid shapes to find their comparison. In the last stage, the subject was unable to complete his explanation.

For students with "newly developing" learning readiness, they have an incomplete geometric construction type because students with "newly developing" learning readiness use geometric solutions and only interiorization, reversal, coordination, and encapsulation mechanisms appear. While the results of the study (Rofiki et al., 2020)Subjects with low mathematical abilities were able to solve cube and cuboid problems. At the process, object, and scheme stages, the subject was unable to complete the cube and cuboid concept indicators. Research (Rosyidi & Hasanah, 2022) on the process of constructing students' new concepts based on gender, the subjects had moderate mathematical abilities. The results showed that male and female students were able to construct new concepts well, although there were some errors that were not fatal. Male students had difficulty discussing their thoughts, while female students were better at conveying ideas orally and in writing.

When compared to previous studies, this study is somewhat different. Previous studies identified students' errors in measuring statistical dispersion using APOS Theory. The findings showed that students with low understanding operate at the action level, while those who are more advanced have reached the schema level (Ng & Chew, 2023). While the results of this study showed that no subjects reached all indicators of this APOS theory. The results of other studies are also somewhat different from the results of this study. Research conducted by Tsafe (2024) shows that most students are still at the Action and Process stages, and have difficulty reaching the Object and Schema stages. This can be seen from the conceptual errors that often appear, such as the inability to connect trigonometric functions with a broader context or to use the function flexibly in problem-solving. Thus, the APOS indicators in this study confirm that students' understanding of trigonometric concepts is not yet fully mature and still requires assistance in order to reach a higher level of schema, which is characterized by the integration and application of more complex concepts. The use of APOS Theory together with the ACE (activities, class discussions, and exercises) learning cycle has proven effective in moving students from the Action and Process stages to the Object and Schema stages. This means that this learning not only helps students perform derivative operations correctly, but also understand concepts in depth and integrate knowledge systematically (Nga et al., 2023).

Other studies have also shown that many pre-service mathematics teachers have limited procedural understanding of injective and surjective functions. This highlights the need for teaching approaches that place greater emphasis on building deep conceptual understanding, so that pre-service teachers can effectively teach these concepts to their students (Bansilal et al., 2017). This is consistent with research conducted by Martinez et. al. (Martínez-Planell & Trigueros, 2019) that many students initially only stay at the Action stage, and have difficulty transitioning to Process and Object.

CONCLUSION

Based on the results of the study, it can be concluded that there is a very strong relationship between learning readiness and concept construction with a coefficient of determination of 90.46% and the rest is influenced by other factors. Students with different learning readiness have different concept construction processes. This is because students' learning readiness is related to initial abilities which are prerequisites for learning or constructing subsequent material. The difference can be seen in the solution to the problems given. Students with "advanced" learning readiness have a complete analytical construction type, students with "developing" learning readiness have a pseudo-complete analytical construction type, and students with "newly developing" learning readiness have an incomplete geometric construction type. The findings of this study suggest that future research should investigate other factors that affect students' concept construction aside from learning readiness. Incorporating additional variables may offer a deeper and more complete insight into the processes by which students build mathematical understanding. Although this study has several limitations, including a limited sample, the results cannot be generalized widely. In addition, the qualitative analysis only used one main question, which may not fully represent the entirety of students' concept construction. This limitation is a consideration for further research with more diverse samples and contexts and more comprehensive instruments. Furthermore, educators are encouraged to tailor their teaching strategies according to students' varying levels of learning readiness. By adopting a more personalized and differentiated approach, the process of concept construction can be enhanced to better meet each student's needs and abilities. In addition, curriculum developers should consider integrating flexible learning approaches and providing diagnostic tools to assess students' readiness to learn. This will allow for customized learning pathways, enabling all students to engage actively and learn optimally in mathematics. These recommendations aim to support the improvement of students' conceptual understanding of mathematics through approaches that are responsive to the diversity in learning readiness and student capabilities.

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