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Investigation of the Mathematical Proof Ability of Prospective Teacher Students Through Cooperative Learning Approach

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Abstract

The research investigates pre-service teachers' ability to create mathematical proofs through cooperative learning, as this skill becomes essential in university mathematics education. The research used a mixed-methods survey design to study 24 students who attended the Mathematics Education program at Universitas PGRI Palembang. Data was collected by a questionnaire, a mathematical proof test, and in-depth interviews. Data analysed descriptively. The results showed that most students were in the medium category of mathematical proof ability using a cooperative learning approach, with smaller proportions in the high and low categories. The investigation concluded that students with high mathematical proof ability demonstrated the ability to implement the five Mathematical Proof Indicators (IPMs) in constructing logical and systematic arguments and using mathematical representations appropriately. Students with moderate mathematical proof ability had difficulty with IPMc (third IPM) and IPMd (fourth IPM), while students with low mathematical proof ability had difficulty with almost all IPMs. The analysis of the investigation results indicates that strategies for conducting mathematical proofs require cooperative learning support to facilitate understanding of the concepts required for proofs and collaboration, and recommends its application in mathematics learning in higher education.

Keywords: ability level; cooperative learning; higher education; mathematical proof; student teacher candidates

INTRODUCTION

A mathematical proof is a valid argument. The understanding and function of proof evolve through the stages of mathematics, culminating in formal proof as understood and accepted by the mathematical community (Hamami & Morris, 2024; Summermann et al., 2021). High school mathematics teachers teach proofs depending on how they define proofs and what they have learned (Ko, Yi-Yin, & Rose, 2022). The complexity of the problems students face in college mathematics differs from that of high school mathematics. At times, proof by example, informal arguments, and similar methods are no longer accepted. Students are required to use axioms and definitions for proof (Almpani & Stefaneas, 2023).

Mathematical proof, which involves deductive reasoning and the use of axioms and theorems, is an important aspect of understanding mathematical concepts in depth (Carl et al., 2021). Furthermore, Yudha et al. (2024) stated that mathematical proofs not only test students' ability to apply the concepts they have learned but also develop critical and creative thinking skills. Students pursuing higher education in mathematics need to develop their ability to write mathematical proofs, as this skill is essential for their academic success (Carl et al., 2021; Nurrahmah & Karim, 2018; G. J. Stylianides et al., 2024). Students need to develop two essential abilities for proof construction: creating valid logical arguments and building critical, analytical, and creative thinking. However, although mathematical proof is a fundamental part of learning mathematics, many students find it challenging to understand and master (Firmasari et al., 2022).

Mathematics at the university level requires prospective mathematics teachers to reach a level of mathematical maturity (Simelane & Engelbrecht, 2023). At this stage, students must not only focus on solving problems with familiar methods and algorithms, but also be able to write proofs, create counterexamples involving abstract objects and concepts, and perform mathematical activities with undefined algorithms (Tupouniua, 2022). Furthermore (Aziz, 2021), the results of his research state that the success or failure of students in providing justification and rationalization is at least influenced by supporting or causal factors, namely: (1) concept understanding, (2) mathematical connections, and (3) mathematical proof. In higher education, especially in mathematics education programs, this ability is crucial because students will later become educators responsible for transferring mathematical knowledge to students at the school level. Therefore, mathematical proof skills need to be improved during college. One way is to familiarize students in mathematics education with the practice of proof. Various studies, both in Indonesia and internationally, show that many students and prospective teachers still struggle to construct proofs logically and systematically (Hartono, 2025). Mathematical proof is widely recognized as an important part of mathematics, yet it is challenging to learn and teach (Almpani & Stefaneas, 2023).

Cooperative learning is efficacious in improving students' understanding and skills, especially in mathematics. In cooperative learning models, students work in groups to help each other, discuss, and solve problems together. This allows them to develop cognitive and social skills simultaneously (Eslit, 2023). Therefore, cooperative learning is a relevant approach to improving mathematical proof skills by providing opportunities for students to learn together, exchange ideas, and deepen their understanding of mathematical concepts (A. J. Stylianides & Stylianides, 2022). Cooperative learning also helps increase students' self-confidence and fosters a more positive attitude towards the learning process in general (Afrison, 2022; Ndebil & Ali, 2024; S. Kaymak et al., 2021).

Mathematical proof skills have become an important focus in higher education, especially for prospective mathematics teachers. (Winer & Battista, 2022) found that many students can explain reasons verbally but have difficulty organizing them into logical, systematic written proofs. (A. J. Stylianides & Stylianides, 2022) emphasize the need for a coherent learning path to help prospective teachers understand the meaning and structure of proofs. Another study by Zazkis & Zaskis (2014) shows that reflective activities, such as lesson play, can help prospective teachers construct an understanding of proofs. Research conducted by Anwar et al. (2023) shows that student teachers demonstrate insufficient ability in proof construction. Research shows that cooperative learning methods enhance student understanding of concepts and their ability to communicate mathematically (Rukmini, D., & Rahardjo, 2021). However, there is limited evidence about its impact on mathematical proof abilities (Rahmawati & Purwaningrum, 2022).

The research focuses on how prospective mathematics teachers develop proof-writing abilities through cooperative learning methods, in contrast to previous studies that focused on conceptual understanding and the development of mathematical communication.

The research by Winer and Battista (2022) studied proof components through oral and written assessments without using specific teaching methods. However, this study implements STAD and Jigsaw as structured cooperative learning models to help students build formal arguments together. The research examines proof development through two methods that combine the evaluation of student-written work with the observation of their group dialogue activities. The research focuses on Indonesian preservice mathematics teachers because it provides a relevant context for developing their teaching abilities and professional competencies (Anwar et al., 2023).

The research introduces a new method that combines cooperative learning techniques with teacher candidate mathematical proof development, with no prior studies on this topic. The research team created a collaborative proofreading task that enables students to work together to develop formal arguments while they learn to analyze their logical reasoning methods. The research uses multiple assessment tools, including written proof evaluation forms, group dialogue observations, and student reflective interviews, to comprehensively measure proof abilities. The research addresses the difference between oral and written reasoning, as identified by Winer & Battista (2022). The research investigates how cooperative learning affects teacher candidates' development of mathematical proofs, drawing on new empirical findings (Rahmawati & Purwaningrum, 2022; A. J. Stylianides & Stylianides, 2022).

This study aims to investigate in depth the mathematical proof skills of prospective student teachers at each achievement level in a cooperative learning context. This area has not been widely explored before. This research is expected to provide new insights into the mathematical proof abilities of prospective teacher students through cooperative learning, mainly by studying these abilities in depth using five indicators of mathematical proof across each ability level, and to offer recommendations for implementing more effective learning in the future.

METHOD

This study, conducted in the Mathematics Education Study Program at Universitas PGRI Palembang, involved 24 participants. The reasons for this were: first, the sample size was sufficiently representative of the student population; second, the relatively small number of respondents allowed for a greater focus on data quality and in-depth analysis (Mthuli et al., 2022).

This study used three instruments:

1. Respondent Attitude Questionnaire toward Cooperative Learning, consisting of three aspects: a) cooperative learning discussion methods; b) respondent activity in the discussion; and c) understanding of the material through discussion learning. This questionnaire is a simple instrument consisting of 15 statements. This instrument is compiled using the "cooperative learning syntax", meaning that the statements or items in the questionnaire are structured around the steps of the cooperative learning approach.
2. The mathematical proof ability test consists of four questions based on school mathematics material, covering geometric transformations, trigonometry, and two number problems. This instrument refers to five indicators of mathematical proof and has been validated by three experts: two mathematical proof validators and one learning evaluation validator.
3. In-depth interviews were conducted with each respondent, who had been grouped into three categories of mathematical proof ability: high, medium, and low (see Table 1 and Table 2). The interviews focused on questions with high response variation and the highest error rates.

The data analysis technique employs a combination of quantitative and qualitative descriptive analysis, as illustrated in Figure 1.

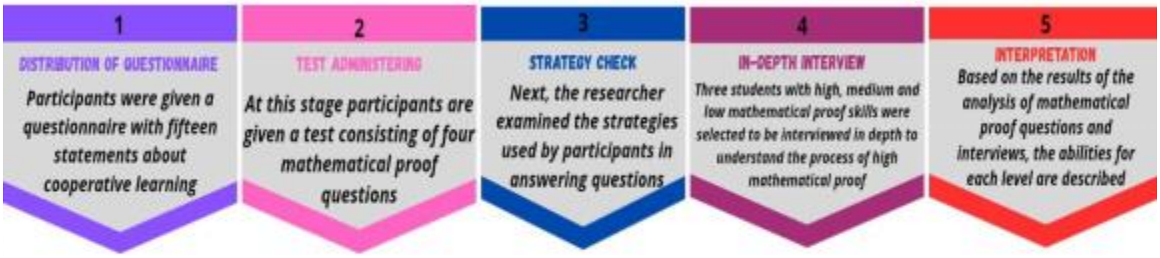


Figure 1. Research Scheme, Modification (Schwartz et al., 2022)

At each stage depicted in the research scheme, the research was conducted to investigate the mathematical proof ability of prospective teacher students through a cooperative learning approach, which can be explained as follows:

1) In the first stage (see Figure 1), the questionnaire comprised 15 statements that measured respondents' attitudes toward cooperative learning experiences in secondary school mathematics courses. These attitudes were classified into three aspects: (1) attitudes toward discussion/cooperative learning methods, (2) active participation in discussions, and (3) understanding of the material through discussions (Ahmed et al., 2020; Alcalá et al., 2019; Mendo-Lázaro et al., 2022)
2) Second Stage: Giving a test to respondents in the form of a posttest with four mathematical proof questions, to obtain a group of mathematical proof abilities (high, medium, and low levels), the results of which will be used to check the strategies used in answering the mathematical proof questions given.
3) The Third Stage of Strategy Checking will be seen from the quality of student work as respondents according to the level of errors made in mathematical proof, which includes 5 indicators of mathematical proof ability, namely a) Understanding the statement to be proven, b) Developing a proof strategy, c) Carrying out the proof process, d) Delivering proof systematically, e) Using the correct representation.
4) The Fourth Stage of in-depth interviews was conducted on 3 prospective teacher students selected based on high, medium, and low levels of mathematical proof ability, in order to understand the mathematical proof process carried out.
5) The Fifth Stage of Interpretation was conducted to obtain an overview of the mathematical proof ability of each level based on the results of the analysis of mathematical proof questions and in-depth interviews about the results of the mathematical proof carried out by respondents on questions with the highest average errors

RESULTS

Results of questionnaire R analysis

The respondents' perceptions play a pivotal role in cooperative learning, as they not only emphasize the achievement of academic learning outcomes but also the development of students' social competence. Consequently, the utilization of these perceptions is highly suitable for learning in higher education (Nurlaila, 2019). The respondents' attitudes towards cooperative learning are evident in the questionnaire results. The data from the questionnaire analysis results were obtained as a whole from respondents' perceptions for 15 statements of Attitudes Towards the Learning Process (STPP). The score conversion with a range of values (0-100) can be seen in Figure 2.

Figure 2. Results of Conversion of Attitude Scores Towards the Cooperative Learning Process

The participants showed a positive attitude toward cooperative learning through their average rating of 75, which falls under the "good" category. The participants demonstrated their complete understanding of cooperative learning through their attitude scores, which reached 100% as shown in Figure 3.

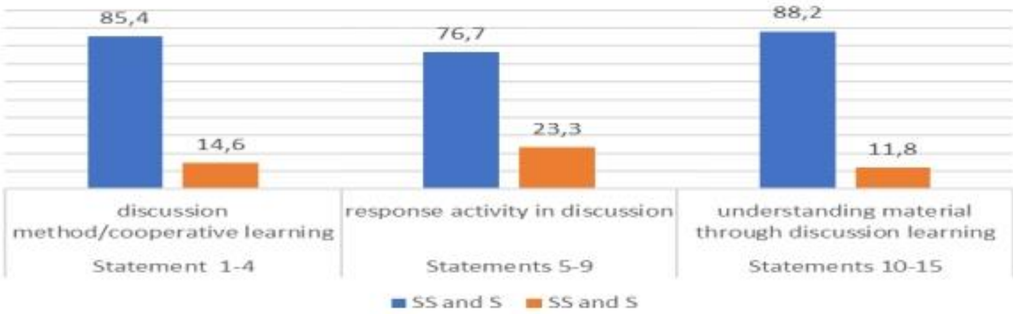


Figure 3. Percentage of Response Scores on 3 Aspects of Attitude in the Cooperative Learning Process

The first section of Figure 3 presents the discussion method/cooperative learning (statements 1–4), which participants praised as an effective learning method that promotes student involvement. The discussion method received positive feedback from 85.4% of participants, while 14.6% disagreed with its effectiveness. According to 85.4% of participants, the Middle School Mathematics Study Course achieved success through its implementation of the discussion method and cooperative learning. The analysis of statements 5-9 revealed that 76.7% of participants actively participated in group discussions and outside group activities, but 23.3% showed limited participation in discussions. The third section on discussion and cooperative learning material comprehension (statements 10-15) received responses from 88.2% of participants, while 11.8% failed to complete their assigned work. The third element of cooperative learning proved to be the most successful method for improving student comprehension. This finding suggests that respondents perceive the discussion method/cooperative learning approach as a facilitator of comprehension of the material in the Middle School Mathematics Study course.

Results of Mathematical Proof Ability Test Analysis

The results of the mathematical proof ability test in this study are presented in Table 1 below.

Table 1. Results of Students' Mathematical Proof Ability Test

No	Descriptive Statistics	Data
1	Questions	4
2	College Students	24
3	Minimum value	23
4	Maximum Value	83

No	Descriptive Statistics	Data
5	Mean	64,71
6	Standard deviation	14,8

Based on Table 1, the mathematical proof ability test results from 24 student respondents yielded an average score of 64.71, categorized as sufficient, with a standard deviation of 14.8. Furthermore, mathematical ability was grouped into three categories, as shown in Table 2.

Table 2. Mathematical Proof Ability Level

No	Level	Test Result (X)	Students
1	High	$x > 79{,}51$	4
2	Medium	$49{,}91 < x \leq 79{,}51$	17
3	Low	$\leq 49{,}91$	3

The Findings of the High School Mathematics Study lecture demonstrate that students' aptitude for proof varies with ability level. The researcher examined the strategies participants used to answer mathematical proof questions. The checking stage revealed the diversity in the quality of students' work, depending on the level of errors made. In summary, the percentage of students' errors in mathematical proof for the four questions is shown in Table 3.

Table 3. Percentage of Mathematical Proof Errors

Code	Indicators analyzed	Error (%) Question number				Average Error Per Indicator
		1	2	3	4	
IPMa	Understanding the statement to be proven.	20,8	20,8	25	33,3	25,0
IPMb	Developing a proof strategy.	54,2	50,3	50	45,8	52,1
IPMc	Carrying out the proof process	29,2	58,3	54,2	62,5	51,1
IPMd	Delivering Evidence Systematically	33,3	58,3	54,2	58	51,0
IPMe.	Using proper representation	21	58,3	50	54	45,8
	Average Percentage Error	31,7	49,2	46,7	50,7	

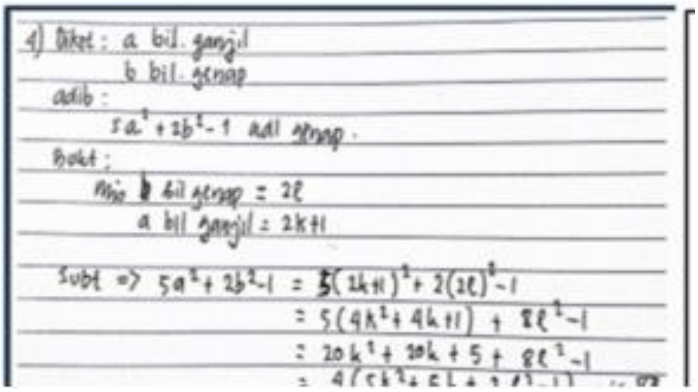
The results of the analysis of proof errors, as presented in Table 3, indicate that respondents demonstrated the highest average proof error on question 4, at 50.7% across the five indicators of mathematical proof ability. The objective of this investigation is to assess prospective teacher students' abilities in mathematical proof using the Cooperative Learning approach. To this end, it is imperative to conduct in-depth interviews with respondents at the high, medium, and low levels regarding the outcomes of their mathematical proofs, as evaluated by the five indicators of mathematical proof.

In the subsequent phase of the study, an interview was conducted with three prospective teacher students. These respondents were selected from each level of mathematical proof ability (M11, M21, and M31) based on Table 2. The objective of the interview was twofold: (1) to obtain a description of cooperative learning support for mathematical proof ability, and (2) to obtain a description of students' mathematical proof ability based on the five indicators of mathematical proof ability.

The ensuing results are the culmination of mathematical proofs conducted by respondents (M11, M21, and M31) with a particular focus on student work in response to question number 4. The aforementioned question concerns verifying the following statement: "If a is an odd number and b is an even number, then $5a^2 + 2b^2 - 1$ is even."

1. High-Level Mathematical Proof Ability (M11)

The answer to M11 is shown in Figure 4.


4) Diket : a bil. ganjil
b bil. genap
adib :
$5a^2 + 2b^2 - 1$ adl genap.
Bukt :
mis b bil genap = 2l
a bil ganjil = 2k+1
Subt => $5a^2 + 2b^2 - 1 = 5(2k+1)^2 + 2(2l)^2 - 1$
$= 5(4k^2 + 4k + 1) + 8l^2 - 1$
$= 20k^2 + 20k + 5 + 8l^2 - 1$

Translation:

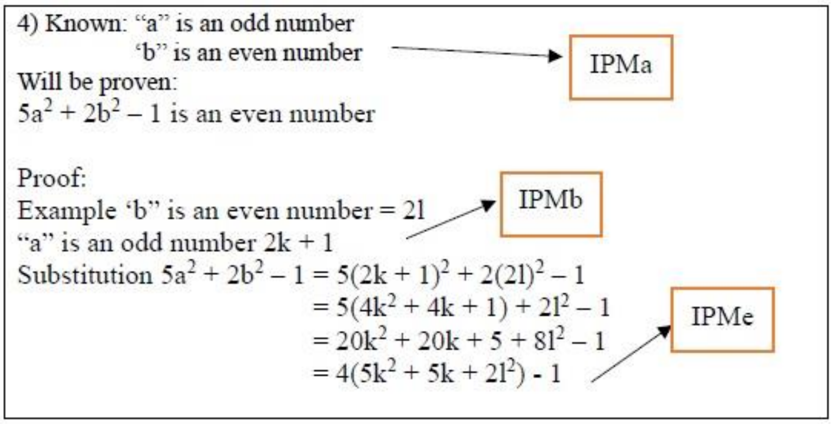


Figure 4. Answer M11 for question no. 4

Understanding the mathematical statement to be proven (IPMa)

Figure 4 illustrates how M11 can comprehend and express mathematical statements more clearly. The subsequent interview script shows how M11.a understood the statement.

Interviewer	:	*Question no. 4 is this: “If a is an odd number and b is an even number, then 5a2 + 2b2 – 1 is even. Why do you assume a = 2l and b = 2k + 1?” Explaining such an analogy, what is in the mind that makes it appear like that?*
M11	:	*To prove that 5a2 + 2b2 – 1 is even. Moreover, because it is known from the question, correct it, ma’am. I meant it like that.*

From the results of an in-depth interview with level 1 students (M 11) regarding their mathematical proof ability test results related to the first mathematical proof indicator (IPMa), it shows that the respondents’ understanding of what will be proven is correct. Their work results demonstrate a strong understanding of restating mathematical proof statements.

Developing a proof strategy (IPMb)

M11 employs a strategy of direct evidence, commencing with the known and progressing to the yet-to-be-proven. This approach is elucidated in the researcher’s interview script with M11 as follows:

Interviewer	:	*Let’s continue with how you strategize to put together this proof?*
M11	:	*Write down what is known and what will be proven.*
Interviewer	:	*Try to provide the proof that is in accordance with question number 4!*
M11	:	*For question number 4, the appropriate type of proof is direct proof, ma’am*
Interviewer	:	*Ooo. So yeah, then..*
M11	:	*I design the next steps by defining each of the knowns.*
Interviewer	:	*Okay, after that, what else?*

M11 : *I carefully operated it after substituting it, and it was finally finished.*
Interviewer : *What is your reason for the proof that was carried out to prove that the statement is true?*
M11 : *That's it, ma'am. In the last line, I wrote the Equation as a multiple of 4, and 4 is an even number; whatever integer is multiplied by an even number, the result will definitely be even.*
Interviewer : *Exactly… your reason is right! (while giving a thumbs up)*

From the results of an in-depth interview with level 1 students (M 11) regarding their mathematical proof ability test results related to the second mathematical proof indicator (IPMb), it shows that the strategy used is sequential from beginning to end. At the end of their work, M11 writes the proof symbol.

Carrying out the proof process (IPMc)

M11, in the course of executing the proof process, articulates arguments sequentially, formulates definitions, and delineates the causal relationships that underpin each step. The interview script is as follows:

Interviewer : Now we will formulate the proof process.
M11 : *Read the question, then write down what is required in this question*
Interviewer : What do you write after reading this question?
M11 : *Wrote the definition of odd numbers and even numbers.*
Interviewer : Furthermore ...
M11 : *Just substitute the definition I wrote into the statement to be proven.*
Interviewer : Yes, so how after being substituted?
M11 : *It's proven, ma'am, I'm confident in the work I've done.*
Interviewer : Have you double checked?

In the interview script, the interviewer seeks to ascertain M11's capacity to articulate mathematical arguments in a logical and coherent manner. In his response, M11 postulates that if a is an odd number, then $a = 2k + 1$, and similarly, if b is an even number, then $b = 2l$, where k and l are natural numbers.

Delivering evidence systematically (IPMd)

Student M11's work on question number 4 has been appropriate, precise, and systematically proven. The following are the results of the in-depth interview script.

Interviewer : *How do you convey evidence systematically?*
M11 : *What do you mean by how do I answer the proof for question number 4?*
Interviewer : *Yes.*
M11 : *In order for my proof to be systematic and correct, I have to read the meaning of this question carefully.*
Interviewer : *How?*
M11 : *Every step I take in answering this question, I have a reason*
Interviewer : *What is the reason in question?*
M11 : *For example, in my mind, "What is the definition of an odd number and what is the definition of an even number?" After I remember, I write it down, then I write down all the necessary mathematical symbols.*

As evidenced by the M11.d interview script, the students have demonstrated a firm grasp of the pertinent concepts and an adept application of these concepts in their responses.

Using proper representation (IPMe)

M11 has employed mathematical symbols correctly to distinguish odd and even numbers. The results of the interview script that was conducted to inquire about "the use of mathematical symbols to strengthen arguments" are presented here.

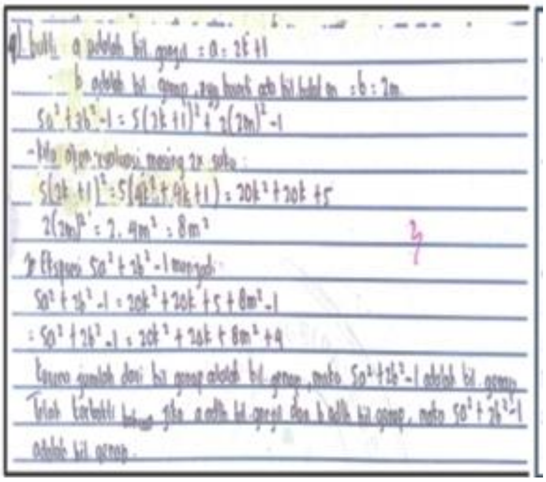
Interviewer : *This is your work, right? It seems like there is no problem. What concepts did you use in this proof?*

M11	:	*Yes, ma'am, this is the result of my work. The concept of numbers, ma'am*
Interviewer	:	*There are odd numbers, and there are even numbers*
M11	:	*Yes ma'am.*
Interviewer	:	*Is there any other concept?*
M11	:	*What other concepts are there, ma'am?*
Interviewer	:	*To lead to proof.*
M11	:	*Oooo Yes ma'am, the concept of multiples is needed and my accuracy in substituting the definition that is assumed to be operated until proven*
Interviewer	:	*That's right (While paying attention to the results of the work M11)*

A thorough review of the M11.e interview script reveals that M11 has demonstrated a comprehensive mastery of all the concepts necessary to accurately represent mathematical symbols.

2. Medium Level Mathematical Proof Ability (M21)

M21 student's answer to question number 4.



4) Proof: "a" is an odd number = a = 2k + 1
"b" is an even number, which means there are "m" integers = b = 2m
$5a^2 + 2b^2 - 1 = 5(2k + 1)^2 + 2\ (2m)^2 - 1$ → IPMa

We will evaluate each tribe
$5(2k + 1)^2 = 5(4k^2 + 4k + 1) = 20k^2 + 20k + 5$
$2(2m)^2 = 2 \text{ x } 4m^2 = 8m^2$

Expression $5a^2 + 2b^2 - 1$ becomes: → IPMb
$5a^2 + 2b^2 - 1 = 20k^2 + 20k + 5 + 8m^2 - 1$
$5a^2 + 2b^2 - 1 = 20k^2 + 20k + 8m^2 + 4$
Because the sum of even number is an even number, then $5a^2 + 2b^2 - 1$ is an even number.
It has been proven that if "a" is an even number and "b" is an even number, then $5a^2 + 2b^2 - 1$ is an even number. → IPMe

Figure 5. Answer M21 Question No. 4

Understanding the mathematical statement to be proven (IPMa)

As illustrated in Figure 5, M21 demonstrated an ability to comprehend straightforward language, thereby facilitating the restatement of mathematical statements. The interview script provides a detailed account of M21's efforts in grasping the statements made.

Interviewer	:	*Let's look at question no. 4. Try reading it! (pwcr shows the existing question script)*

M21 : *M21 examines question number 4 in the question paper.*
Interviewer : *If a is an odd number, a = 2k + 1, and b is an even number, then b = 2m. The writing is M21 (pwcr mentions the student's name). Is this correct??*
M21 : *Yes ma'am*
Interviewer : *Is that all? (while showing the answer sheet from M21)*
M21 : *Yes ma'am.*
Interviewer : *There is no continuation here, right? Let's look closely at what will be proven in this question.*
M21 : *Oooo Yes ma'am, I forgot to write what I was going to prove*
Interviewer : *Yes, it is better to write down what is known and what is asked/proven in the problem to solve it.*

In the M21 interview script, M21 admitted that during the proof, he neglected to document the intended demonstration because he was elated to have successfully derived the symbols for odd and even numbers.

Developing a proof strategy (IPMb)

M21 endeavours to employ a direct proof strategy, commencing with the processing of the known elements of the problem and culminating in the articulation of the intended proof. M21's work will delineate proof steps; however, the text is direct, resulting in illogicality, manifesting as leaps of thought. This is elucidated in the interview script as follows.

Interviewer : *Forgot! (while showing M21's work). After that, there must be a sentence of what will be proven.*
M21 : *Yes ma'am.*
Interviewer : *Until the substitution of the numbers a and b in what will be proven is correct. Why must there be the word "evaluation"? What does that mean?*
M21 : *The point is that the right side of the proof is separated for its operations to be more detailed, but the reality really makes me confused, ma'am.*
Interviewer : *So what is the truth?*
M21 : *No need to separate them.*
Interviewer : *Okay, what next?*
M21 : *I saw that my work was not in line with the strategy I wanted.*

M21's employment strategy is limited to interpreting written language, offering no revision of the error. After the interviewer transitions to the next topic, M21 can propose the next proof strategy.

Carrying out the proof process (IPMc)

The logical and coherent presentation of mathematical arguments is paramount for the proof process, and M21's work exhibits deficiencies in this regard. As articulated in the response, under the assumption that a is an odd number, it is expressed as $a = 2k + 1$, and that b is an even number, it is written as $b = 2m$. The result is then substituted into the number to be proven; however, in the solution, it is separated and less precise. Ultimately, the definition of an even number is not simplified to prove the final proof.

Delivering evidence systematically (IPMd)

Student M21's work on question 4 does not align with the established proof. The student did not present a systematic proof. During the interview, M21 demonstrated confusion while writing the storyline, which hindered their ability to produce a systematic proof. The following is a transcript of the interview that highlights the student's challenges with question 4.

Interviewer : *Starting from the known. odd numbers and even numbers. What was on your mind at that time, so that the next job was like that?*
M21 : *(silence for 5 seconds) At that time, I saw the numbers that were to be proven, and I immediately substituted odd and even numbers.*
Interviewer : *Yes... that's right, that's it. But why is the next step like this... (by looking at the results of M21's work)*
M21 : *Yes, ma'am, why do I do that?*

Interviewer	:	*Do you understand now?*
M21	:	*Yes, ma'am, ehm......so embarrassed.*

As illustrated in the M21.d interview script, M21 continues to face challenges in carrying out systematic proof. This means that IPM d. was not successfully implemented.

Using proper representation (IPMe)

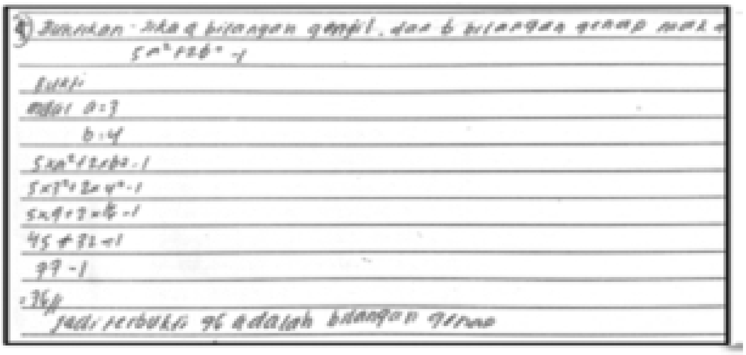
M21 has correctly used mathematical symbols to define odd and even numbers. However, an inaccuracy has been identified in the proof's use of numbers. The results of the M21.e interview script, which was conducted to inquire about "the use of mathematical symbols to strengthen arguments," are presented here.

Interviewer	:	*In question number 4, what concepts are related?*
M21	:	*(Silence in 5 seconds) Numbers ma'am*
Interviewer	:	*What numbers?*
M21	:	*Odd numbers and even numbers*
Interviewer	:	*Do you know the definition of odd and even numbers?*
M21	:	*Yes ma'am. I understand the definition.*
Interviewer		*Try to name it!*
M21		*Odd numbers are.... and even numbers are....*
Interviewer		*Ok, good answer. But why is the solution made like this (while pointing to the results of M21's work)*
M21		*Yes, ma'am, why did I write this? M21 with a confused face.*
Interviewer		*Thank You*

The M21.e interview script shows that students understand what odd and even numbers are and how to create symbols for proof development. The students showed signs of confusion after this point. The in-depth interview with M21 showed his positive attitude, while he successfully identified his numerical mistakes and carelessness.

3. Low-Level Mathematical Proof Skills (M31)

M31 student's answer is shown in Figure 7.



Prove if "a" is an odd number and "b" is an even number then $5a^2 + 2b^2 - 1$
Proof:
Example: a = 3
b = 4
$5 \times a^2 + 2 \times b^2 - 1$
$5 \times 3^2 + 2 \times 4^2 - 1$
$5 \times 9 + 2 \times 16 - 1$
$45 + 32 - 1$
$77 - 1$
76
So it is proven that 76 is an even number

Figure 7. Answer M31 Question No. 4

Understanding the mathematical statement to be proven (IPMa)

As illustrated in Figure 7, M31's comprehension of the proof's necessity is limited to its application to specific examples, rather than to its general principles. The statement "if a is an odd number and b is an even number, then $15a^2 + 2b^2 - 1$ is an even number" necessitates proof that extends beyond its application to specific values such as a = 3 and b = 4. The interview script M31.a indicates that M31's approach to the proof is not universally applicable.

Interviewer	:	*Let's look at question no. 4. Try reading it. (Interviewer shows the question paper on M31)*
M31	:	*Pay attention to question number 4 in the question paper.*
Interviewer	:	*If a is an odd number, a = 3, and b is an even number, then b = 4. The writing on M31 (pwcr mentions the student's name) is this correct?*
M31	:	*Yes ma'am*
Interviewer	:	*Why don't you first write down what is known and what is proven?*
M31	:	*I have written it down, ma'am. For example, a = 3 and b = 4. And I forgot the definition of odd and even numbers. I only remember even and odd numbers, but I forgot the definition.*
Interviewer	:	*Oooo, so yeah. It must be remembered that if you want to prove it, you need an algebraic argument that shows that the property applies in general, not just to certain pairs of numbers.*
M31		*Yes ma'am*

In the interview, M31 stated that he currently lacks the ability to provide a mathematical proof. His confession indicates a potential lapse in his understanding of the fundamental concepts of odd and even numbers, yet he demonstrated an ability to recall specific case examples, which he perceived as sufficient for the proof.

Developing a proof strategy (IPMb)

M31 does not explicitly write the properties of a and b (odd-even) in algebraic form.

For example:

i. odd numbers can be expressed as a = 2k + 1 where k is an integer.
ii. Even numbers can be expressed as b = 2m, where m is an integer.

The following is the M31 interview script.

Interviewer	:	*Now let's look at proof question number 4. (while mentioning the name M31 and showing the results of his work) Did you do it like this?*
M31	:	*Yes ma'am*
Interviewer	:	*Pay attention to your questions and answers (the interviewer mentioned the name M31); we will discuss it later.*
M31	:	*Pay attention for 8 seconds.*
Interviewer	:	*So, this is new?*
M31	:	*Yes ma'am*
Interviewer	:	*What happened at that time? So, the answer (the interviewer mentioned the name M31) was limited to that.*
M31	:	*I forgot and....*

The findings of the job analysis and interviews with M31 suggest that students with inadequate mathematical proof skills encounter challenges in formulating effective strategies. This phenomenon is characterized by an inability to apply knowledge from one problem to another, indicating a deficiency in problem-solving skills.

Carrying out the proof process (IPMc)

The M31 error in the proof process stems from the utilization of a single example to infer the universal validity of a statement. This approach constitutes a logical fallacy, as verifying a particular assertion does not inherently substantiate its broader applicability.

Delivering evidence systematically (IPMd)

It has been established that the findings of the work and interviews of Student M31, concluded directly from a single example without the support of algebraic or logical arguments, demonstrate the applicability of the property to all cases. The recent interviews with the new student participants have demonstrated that conclusions must be founded on formal arguments. Subsequent algebraic manipulation reveals that the results are constant for all odd and even values of *b*.

Using proper representation (IPMe)

M31 performs numeric value substitution without utilizing mathematical representation that reflects the properties of odd and even numbers. The results of the interview script that has been implemented to ascertain the appropriate representation are as follows:

Interviewer	:	*Let me ask M31 (the interviewer mentioned M31's name) to tell you: when faced with a question like this, what appears on the question paper?*
M31	:	*(Silence for 7 seconds) Numbers ma'am*
Interviewer	:	*What numbers?*
M31	:	*Odd numbers and even numbers*
Interviewer	:	*Do you know the definition of odd and even numbers?*
M31	:	*No ma'am, I forgot.*
Interviewer	:	*Okay, try to make the general form of odd numbers? Or the general form of even numbers? (with hints)*
M31	:	*Eeeee.... (Silent)*
Interviewer	:	*Well, it's almost correct for the general form of even numbers*
M31	:	*Yes ma'am, if an even number is divided by 2, there is no remainder, so I just remembered that the definition of an even number is......*
Interviewer	:	*Well, go ahead.*

A review of the interview script reveals that M31 has yet to fully grasp the distinction between odd and even numbers. Through in-depth interviews and the interviewer's guidance, M31 has come to understand that to substantiate his claims, he must employ assumptions formally rather than relying on specific examples to ensure the veracity of his assertions.

As demonstrated in Table 3 and elucidated through in-depth interviews with the three respondents, who serve as proxies for the three levels of mathematical proof ability, the analysis substantiates the conclusions presented in Table 4.

Table 4. Results of Mathematical Proof Ability Analysis

MPA	Respondents	Mathematical Proof Indicators				
		IPMa	IPMb	IPMc	IPMd	IPMe
High	M11	√	√	√	√	√
Medium	M21	√	√	×	×	√
Low	M31	√	×	×	×	×

As illustrated in Table 3, the outcomes of the analysis pertaining to students' mathematical proof abilities find corroboration in the results of in-depth interviews, with IPMa (understanding mathematical statements to be proven) attaining 100% and IPMb (developing proof strategies) reaching 66.7%, for IPMc (Conducting proof processes) 33.3% and for IPMd (Conveying proofs systematically) 33,3%, as well as 66.7% for IPMe (Using appropriate representations). The findings reveal no discrepancy in students' understanding of mathematical statements to be proven across high, medium, and low levels of mathematical proof ability (Mujib & Firmansyah, 2022). However, significant disparities emerge when students are tasked with solving problems related to the indicators of conducting proof processes and conveying proofs systematically. Pre-service teachers, in particular, with low levels of mathematical proof skills, faced challenges in completing the other four indicators. Conversely, pre-service teachers with high and medium levels of mathematical proof skills demonstrated the ability to understand statements, develop strategies, and use appropriate representations

DISCUSSION

The research findings demonstrate that students who participate in cooperative learning discussions during the discussion method develop a better understanding of mathematical concepts in their high school classes. Students who work together in Cooperative Learning environments help each other while sharing responsibility for achieving group success (Flaherty, 2022; Wattanawongwan et al., 2021). Students who use this learning approach develop their analytical abilities when they solve mathematical problems that require systematic and logical thinking (Imjai et al., 2024; Schwartz et al., 2022). The educational approach of Cooperative Learning helps students build social abilities through teamwork and self-assurance, and improve communication skills (Ruijuan et al., 2023).

Students' mathematical proof abilities develop through Cooperative Learning, which provides group work and active learning activities that help them develop proof strategies, present logical mathematical arguments, and use correct symbolic notation. Research studies (Silva et al., 2023; Marco N., 2022) support this claim by showing that Cooperative Learning helps students develop advanced thinking abilities, solve problems and communicate effectively. Students who work together in Cooperative Learning environments use critical thinking to solve problems. The research conducted by Mohammed Alharbi et al. (2022) supports this finding through the Sociocultural Learning theory framework. The theory shows that students learn better through social interactions with peers when working in groups. Students who work in groups learn mathematical proofs better because more-skilled group members provide support during their learning process.

Research indicates that implementing cooperative learning significantly impacts mathematical proof skills (Ndebil & Ali, 2024). The method encourages students to engage in systematic discussions of problem-solving strategies, facilitating their understanding of the logical flow and enhancing their proficiency in formulating valid arguments. It is important to note that proof is not merely a technique for proving mathematical truths; it is also a tool for developing abstract and logical thinking skills. Through exercises and discussions in cooperative learning, students can develop problem-solving skills (Tupouniua, 2022).

The examination of students' strategies revealed multiple incorrect definitions that served as essential proof requirements. The research results match the findings from previous studies (Baugerud et al., 2025). Students failed to answer the prompts because they had forgotten the proof structure, even though they had worked with mathematical proofs in their previous educational materials. The research by Nurrahmah and Karim (2018) showed that students struggled to solve proof problems, as 60% of participants failed to understand the solution methods. The application of high-level thinking through mathematical proof requires students to demonstrate complete understanding of fundamental concepts (Himmah, 2022; Lusiana & Kesumawati, 2023; Poggiolesi & Genco, 2021).

The Cooperative Learning method helps students achieve academic success while building social skills (Nurlaila, 2019). The method works best in higher education because it supports student-centred learning through instructor facilitation, helping students develop knowledge and skills by working together in groups. The method supports diverse student needs by creating learning opportunities that are fair for all members of each group. The implementation of Cooperative Learning in educational settings creates an optimal learning environment through student-to-student interactions (Pangantihon & Tantiado, 2024). external help. The method enables students to develop independence throughout the course by making them responsible for their learning targets without needingexternal assistance.

The research results show that students who struggle to develop basic mathematical proofs exhibit three main characteristics. Students face obstacles when translating complex mathematical statements into everyday language while preserving their original meaning. Students face obstacles when they need to create structured proof plans because they struggle to establish starting points and proof objectives, and to maintain logical connections in their arguments. Students face regular problems when organizing their proofs which results in disconnected and less effective written work. The proofs fail to maintain a structured format because they do not separate the beginning assumptions from the main evidence and final results. Students make mistakes when writing proofs because they use incorrect symbols and non-standard notation, which creates confusion and leads to incorrect interpretations of the proofs. Students need to study mathematical concepts in depth to succeed in creating mathematical proofs. The process of creating proofs requires students to understand mathematical concepts deeply because proofs require both technical skills and the ability to link mathematical ideas into logical sequences (Aisyah et al., 2023; Poggiolesi & Genco, 2021).

CONCLUSION

From the results of the implementation of the study consisting of 5 stages in conducting an investigation of the mathematical proof ability of prospective teacher students through the Cooperative Learning approach, it was concluded that from the results of the student attitude questionnaire towards the cooperative learning process, it was concluded that respondents felt that learning with the discussion method/cooperative learning could help them understand the material of the Middle School Mathematics Study course. Based on the study results, the mathematical proof ability of prospective teachers through the cooperative learning approach had an average of 64.71, a standard deviation of 14.8, and 16.7% high ability, 70.8% medium, and 12.5% low. Furthermore, from the results of in-depth interviews with respondents at the three levels of mathematical proof ability related to all Mathematical Proof Indicators (IPM) that appeared in the mathematical proof process carried out, it was found that at the high level of mathematical proof ability, it showed the ability to implement the 5 IPM (Mathematical Proof Indicators) in compiling logical, systematic arguments, and using mathematical representations correctly. At the medium level, they have difficulty with IPMc (Conducting the proof process) and IPMd (Delivering proof systematically), while the low level has difficulty with almost all IPMs. Based on the analysis of the investigation findings, it can be concluded that the strategy for conducting mathematical proof requires the support of Cooperative Learning to facilitate understanding of the concepts needed in proof and collaboration, and recommends its application in mathematics learning in higher education. This suggests that Cooperative Learning can be an effective approach to improving mathematical proof skills. However, greater attention is needed to support low-ability student teachers to overcome obstacles in performing mathematical proofs.

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