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Designing a PMRI-Based Learning Trajectory for Algebraic Forms Using the Iftar Context

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Abstract

Algebra is an essential part of mathematics and is still challenging for students to understand, including algebraic forms. Difficulties concern mathematization, understanding algebraic expressions, applying arithmetic operations in numerical and algebraic expressions, understanding the concept of variables, and formulating mathematical models. The design of a Hypothetical Learning Trajectory (HLT) with PMRI learning principles that integrate the context of Iftar can assist students in understanding this algebraic form. This study aims to describe the design and implementation of HLT for learning algebraic form using the context of Iftar. The method used was design research with validation studies, consisting of three main phases: preparing for the experiment, experimenting in the classroom (pilot and teaching experiment), and conducting retrospective analysis. The subjects were 18 7th-grade students of MTs Laboratorium UIN Sultan Thaha Saifuddin Jambi. Data were collected through observation, interviews, and documentation. The results showed that learning using the context of Iftar could promote students' understanding of algebraic form, students stated a situation in algebraic form, used algebraic operations in problem solving, and provided real experience solving problems with algebra.

Keywords: algebraic forms; design research; iftar; PMRI; realistic mathematics education

INTRODUCTION

The Indonesian curriculum has regulated the curriculum structure, including intracurricular (Kemdikbud, 2024). Based on the learning achievements of phase D of junior high schools (BSKAP, 2024), among the learning achievements are that students can express a situation in algebraic form and use the properties of operations (commutative, associative, and distributive) to produce equivalent algebraic forms. Algebra is often considered the 'gatekeeper' in school mathematics, which is very important for further study in mathematics and future educational and employment opportunities (Cai et al., 2011; Nataraj & Thomas, 2016). Learning algebra in schools is important because it introduces students to mathematics by modelling relationships and handling abstract quantities (Veith et al., 2023).

However, there are still obstacles to learning algebra. The main difficulties students face in early algebra involve mathematization, understanding algebraic expressions, applying arithmetic operations in numerical and algebraic expressions, understanding the different meanings of the equal sign, and understanding the concept of variables (Jupri et al., 2014; Noto et al., 2020; A. M. J. Putri et al., 2020). Students often face epistemological barriers due to limited knowledge of algebraic concepts, such as variables, coefficients, and constants (Beeh et al., 2018). This lack of understanding hinders their ability to interpret and manipulate algebraic forms (Dhlamini, 2023; Nansiana et al., 2024; Noto et al., 2020). Another challenge is the transition from arithmetic to algebraic thinking (Chasanah et al., 2015; Kieran, 2018; Wahyuni et al., 2023).

About solving story problems, common errors that appear are transformation errors, namely, students cannot interpret or change text into mathematical statements (Kenney & Ntow, 2024) or formulate mathematical models, as evidenced by errors in formulating equations, schemes, or diagrams (Jupri & Drijvers, 2016). Many errors in understanding algebra are caused by teaching methods and students' intuitive reasoning. Effective teaching can overcome this problem, suggesting that educators consider the initial presentation of algebraic concepts (Macgregor & Stacey, 1997). To overcome this obstacle, the Indonesian Realistic Mathematics Education (PMRI) approach that uses real-life contexts can improve modelling, reasoning, representation (Pitriani, 2016; Purwitaningrum & Prahmana, 2021), and problem-solving skills (Bouck et al., 2025; Khairunnisak et al., 2021).

Learning with the PMRI approach integrates three design principles in guiding the learning process: Guided reinvention, didactical phenomenology, and emergent models (Doorman et al., 2016). Teachers organize class discussions to help students develop mathematical ideas in guided reinvention. This approach involves a significant shift in authority from teachers to students, encouraging them to take responsibility for their learning (Solomon et al., 2021). Didactical phenomenology is a meaningful context in students' lives through experiences to encourage the development of mathematical objects that students learn (Doorman et al., 2016). With self-created models guiding students through the process of abstraction, where they start with concrete and realistic problems and gradually develop more formal mathematical reasoning (Gravemeijer, 2020).

Several studies have been conducted in designing algebra learning, including on the material of arithmetic sequences and series (Andzin et al., 2024; Domu & Mangelep, 2020), number patterns (Yulia et al., 2020), multiplication of algebraic forms (Yuliani et al., 2024), and linear equations of two variables (Agustina & Zulkardi, 2020; Eriyenti Putri, 2019; M. R. P. Utami et al., 2024). Using several contexts, including Borobudur temple (Andzin et al., 2024), lamp decoration (Yulia et al., 2020), matches and ice cream cups (Domu & Mangelep, 2020), rice fields, commemoration of Indonesian Independence Day, fish ponds, ceramic installation (Yuliani et al., 2024), shopping for stationery (Agustina & Zulkardi, 2020), and Jakabaring tourism (M. R. P. Utami et al., 2024). PMRI can help students understand concepts (Andzin et al., 2024; Domu & Mangelep, 2020; Eriyenti Putri, 2019; M. R. P. Utami et al., 2024; Yuliani et al., 2024), student reasoning (Yulia et al., 2020), and problem solving (Agustina & Zulkardi, 2020).

Previous research shows that PMRI learning design can support students in understanding algebraic concepts, reasoning, and problem solving. However, there is a gap in research regarding PMRI-based learning design that combines cultural and religious activities in supporting students' ability to express a situation in algebraic form and problem solving. The novelty of this study lies in the intervention of designing a Hypothetical Learning Trajectory (HLT) using the activity of iftar, with the problem of money paid or spent when eating at a restaurant, during iftar. This problem allows students to express a situation in algebraic form, perform algebraic operations, use the properties of algebraic operations, and solve problems in everyday life using algebra. This study aims to describe the design and implementation of HLT algebra learning to express a situation in algebraic form and solve problems in the context of everyday life using algebra through the activity of iftar and contribute to the Local Instructional Theory (LIT) consists of assumptions that support the learning process and productive student activities.

METHOD

This study uses a validation study type of design research to design a learning trajectory on algebraic form material with the PMRI approach using an iftar context for seventh-grade students. This context was chosen because the students came from Madrasah Tsanawiyah, which is relevant to Iftar restaurant activities with family or friends. This study was conducted from March to April 2025. Validation study aims to design a learning trajectory by testing learning theory through HLT (Nieveen et al., 2006). This process consists of three main phases: (1) preparing for the experiment, (2) experimenting in the classroom, and (3) conducting retrospective analysis (Gravemeijer & Cobb, 2006). Six students participated in the Pilot experiment, and 18 7B grade students selected by subject teachers became the main research subjects during the teaching experiment.

The preparation for the experiment phase aims to formulate a local learning theory that is elaborated and refined during the experiment process. In this phase, a series of activities, including determining learning objectives, student activity sheets, and students' thinking assumptions, are developed in the Hypothetical Learning Trajectory (HLT). Through FGD activities, to get input from experts and practitioners. The experimenting in the classroom phase aims to explore and observe students' strategies and thinking. There are two cycles in this phase, namely: (1) Pilot experiment: The purpose of this cycle is to improve the quality of the HLT designed in the initial stage through small class learning activities consisting of six students; (2) Teaching experiment: The purpose of this cycle is to test the HLT after being adjusted during the pilot experiment, as well as to obtain actual data on the improved HLT.

The conducting retrospective analysis phase aims to analyse all data collected in the learning experiment by comparing the HLT prepared and the Actual Learning Trajectory (ALT). The results of the retrospective analysis will be used to develop the HLT until it is considered sufficient to produce a better and more relevant LIT. Data were collected through observation, interviews, and documentation. Qualitative analysis involves triangulation, using various data sources, methods, or perspectives to cross-check and validate findings, and ensure their reliability and accuracy. Triangulation is carried out by collecting various data sources, such as observations, interviews, documentation, and student worksheets, through techniques such as FGD, pilot experiments, and teaching experiments, which involve collaboration between researchers, teachers, and experts in analyzing data to minimize subjective bias.

RESULTS

Preparing for the experiment

Before designing student activity sheets, a literature review was conducted on algebra learning using the PMRI approach and learning outcomes and indicators in the Merdeka curriculum. PMRI uses contexts directly felt by students (R. I. I. Putri et al., 2015), and algebra learning related to solving contextual problems can use several strategies, including drawing diagrams, forming equations, and making tables (Kwabena Tuffour, 2014). The context of iftar choosing is because it is very close to students' daily lives during Ramadan, so it is hoped that students will be motivated to carry out learning activities.

Table 1. HLT of Learning Algebraic Forms with the Iftar Context.

Idea	Activity	Conjecture
1. Observing the Menu 2. Discussing Experiences 3. Discussing what influences the amount of money paid. 4. Stating the problem in algebraic form 5. Identifying algebraic forms 6. Writing algebraic forms	1. Students observe the menu of the Meriah Sambil Lahap package. 2. Students discuss the experience of iftar at a restaurant. 3. Students discuss what influences the amount paid for iftar at a restaurant. 4. Students try to state the problem in algebraic form using a table of component relationships that affect the money paid or spent when eating at a restaurant. 5. Students identify the changing values (variables), their multiplying factors (coefficients), and the constant values (constants). 6. Students write the algebraic form of the money paid or spent for iftar at a restaurant.	1. Students can find out the menu of the Meriah Sambil Lahap package. 2. Students share their experiences of iftar at a restaurant. 3. Students find that the number of family members who join, the food package price, additional tax costs, and parking affect the money paid. 4. Students can state the problem in algebraic form using a table (number of family members x package price + tax + parking). 5. Students can determine the values that change (number of family members and number of people who join), the multiplying factor (price of each package), and the fixed values (tax and parking). 6. Students write the algebraic form of the money paid or spent to eat at the restaurant: the number of family members x package price + tax + parking fee, and the Number of members x package price at each table + tax + parking fee according to the number of motorbikes.
7. Performing algebraic operations 8. Using the properties of algebraic operations	7. Students perform algebraic operations to obtain the money paid or spent for iftar at a restaurant. 8. Students use the properties of operations to simplify the algebraic form to obtain the money paid or spent when having iftar at a restaurant.	7. Students perform algebraic operations of addition, multiplication, and division to obtain the money paid or spent for iftar at the restaurant. 8. Students use the distributive property to simplify the algebraic form of money paid or spent when having iftar at a restaurant. Students do not use the distributive property to simplify the algebraic form of money paid or spent when having iftar at a restaurant.
9. Solving problems	9. Students solve problems regarding money paid or spent when having iftar at a restaurant.	9. Students can solve problems regarding money paid or spent when having iftar at a restaurant, but some students cannot.

Based on the literature review, the learning objectives chosen are that students can express a situation in algebraic form and can use the properties of operations (commutative, associative, and distributive) to produce equivalent algebraic forms and solve problems in the context of everyday life using algebra.Using the PMRI approach with the context of iftar, the HLT prepares as shown in Table 1 to guide students to achieve learning objectives through contextual activities in 3 sets of activity sheets: activity 1, activity 2, and activity 3. Before the small group trial, an FGD was conducted with experts and practitioners to revise the draft of the student activity sheet. The results of the revision are shown in Table 2.

Table 2. FGD Results

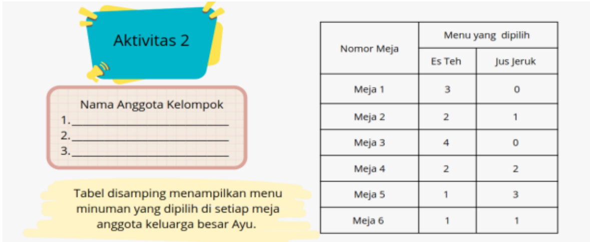
Suggestions	Revision
Replacing the word “outside” with “at the restaurant” in activity 1, number 1.	The word “outside” has been replaced with “at the restaurant”.
In activity 1, number 3, replace the word “influencing” with “causing,” add the word “must,” and replace the word “eating out” with “at the restaurant”.	The word “influencing” has been replaced with “causing”, the word “must” has been added, and the word “eating out” has been replaced with “at the restaurant”.

Suggestions	Revision
Replacing one crushed chicken menu on table 1 with catfish pecel and two crushed chicken menus with catfish pecel on table 3, so that there is a difference in the price of the menus on the table in activity 3.	One crushed chicken menu on table 1 has been replaced with catfish pecel, and two on table 3 have been replaced with catfish pecel.
Creating an auxiliary table to help express the situation of three tables to make it easier for students to express it in algebraic form in activity 3, number 13.	An auxiliary table to express the three-table situation has been created to make it easier for students to express it in algebraic form.

Experimenting in the classroom

Pilot experiment

The pilot experiment was conducted in two sessions, each lasting 80 minutes, involving six students randomly selected from class 7 MTs Laboratorium UIN Sulthan Thaha Saifuddin Jambi. The purpose of this cycle is to improve the quality of HLT designed in the initial stage through small class learning activities. The student activity sheet designed with the context of Iftar is shown in Figure 1 below.



Nomor Meja	Menu yang dipilih	
	Es Teh	Jus Jeruk
Meja 1	3	0
Meja 2	2	1
Meja 3	4	0
Meja 4	2	2
Meja 5	1	3
Meja 6	1	1

Translated into English:
The table beside shows the drink menu chosen at each table of Ayu's extended family members.

(a) (b)

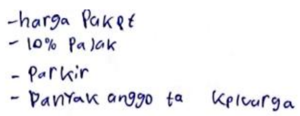
Translated into English:
During Ramadhan, Ayu's family wants Iftar at the Sambil Lahap restaurant. Her family consists of her parents, one older brother, one younger brother, and herself. From home, Ayu's family drives a car with a parking fee of Rp. 5,000. State the algebraic form of the amount of money that Ayu's parents have spent when returning home after Iftar! Then determine the amount of money that has been spent!

(c)

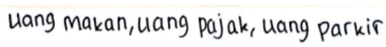
Figure 1. Context of Iftar Activities

Discuss what influences how much money is paid

Students observe the set menu and discuss the experience of Iftar at a restaurant, then identify what influences the amount of money paid. This activity aims to help students identify relevant information influencing how much money will be paid for Iftar at a restaurant. Based on Figure 2, in answer (a), the student has completely identified everything that influences the amount of money paid; however, in answer (b), it is not complete, only three of the four things that influence it. Therefore, guided observation and identifying information from the context are needed.



- package price
- 10% tax
- Parking
- family members



Food cost, tax cost, parking fees

(a) (b)

Figure 2. Example of Student Answers Identifying Things That Influence the Amount of Money Paid

Stating the problem in algebraic form

This activity aims to enable students to state a situation in an algebraic context using the help of tables or visual models. Based on Figure 3, students can state a situation in a context with no additional costs and only one additional parking fee, in (a), students try to state in the form of variables a, b, and p, respectively, to represent the price of smashed chicken, grilled chicken, and catfish pecel. Students still have difficulty stating a situation in the context when there are two additional costs in a row, tax and parking costs together in one algebraic form (b), stating additional tax costs only, writing 10%. Therefore, guidance is needed by providing additional tax information and calculating tax costs in food shopping transactions.

Meja	Jumlah	Menu	Harga	Ekspresi Aljabar
1	2	ayam penyet	48.000	2a + p
	1	pecel lele	29.000	
2	2	ayam penyet	48.000	2a + b
	1	ayam bakar	29.000	
3	2	ayam penyet	49.000	2a + 2p
	2	pecel lele	58.000	

(a)

Banyak anggota keluarga	Harga Paket	tambahan biaya pajak	Tambahan biaya Parkir Mobil	Ekspresi Matematika	Jumlah uang yang dikeluarkan
1	24.000	10%	5.000	(1x 24.000)+10% (24.000) + 5000	26.400
2.	24.000	10%	5.000	(2x 24.000)+10% (2x24.000) +5000	52.800
3.	24.000	10%	5.000	(3x 24.000)+10% (3x24.000) +5000	79.200
4.	24.000	10%	5.000	(4x24.000+10% (4x 24.000) +5000	102.600
5.	24.000	10%	5.000	(5x 24.000+ 10% (5x24.000) +500	132.000

(b)

Figure 3. Example of Student Answers Stating the Problem in Algebraic Form

Identifying algebraic forms

This activity aims to enable students to identify changing values (variables), multiplying factors (coefficients), and constant values (constants) by providing guidance using additional information contained in the activity sheet. Based on Figure 4, students can identify changing values, multiplying factors, and fixed values based on the algebraic form of the previous activity (b). However, some students still have difficulty identifying it (a). Therefore, guidance is needed by observing and identifying information in the algebraic form column from the previous activity, as well as the additional information provided.

Nilai yang berubah: Banyak anggota keluarga dan ekspresi mate.
Faktor pengali dari nilai yang berubah: Banyak anggota keluarga dan harga
Nilai yang tetap: tambahan biaya parkir mobil

Changing value: Family members and mathematical expressions
Multiplier: Family members and package prices
Constant value: Additional car parking fees

(a)

Nilai yang berubah: Banyak anggota kelua
Faktor pengali dari nilai yang berubah: Harga pa
Nilai yang tetap: Biaya parkir

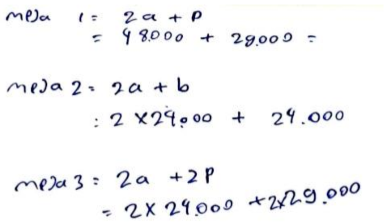
Changing value: Family members
Multiplier: Package prices
Constant value: Parking fees

(b)

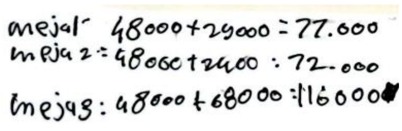
Figure 4. Example of Student Answers Identifying Algebraic Forms

Writing algebraic forms

This activity aims to enable students to write algebraic forms containing changing values (variables), multiplying factors (coefficients), and fixed values (constants) of a situation in the context of money paid or spent when Iftar at a restaurant. Based on Figure 5, students can write the algebraic form of a situation in the context with additional parking fees only and with additional tax and parking fees. Some students add up their money directly in addition to the algebraic forms (a) and (b). Some students try to write algebraic forms in three variables, a, b, and p, respectively, to represent the price of smashed chicken, grilled chicken, and catfish pecel (b). Some students cannot write the algebraic form of a situation in context. Therefore, it is necessary to be guided by observing and identifying information in activity number 14, writing algebraic forms on each table and relating them to the algebraic form of money needed with additional tax and parking costs, as well as providing additional information about taxes and calculating tax costs in food shopping transactions and using brackets to group expressions in algebraic order to make it easier to see which algebraic operations are performed first.



(a)

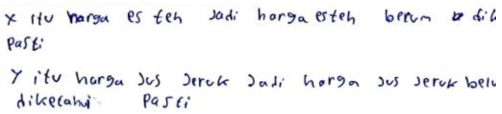


(b)

Figure 5. Example of Student Answers Writing Algebraic Forms

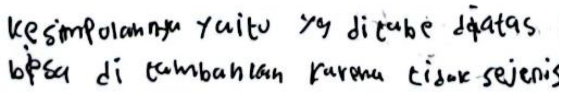
Performing algebraic operations

This activity aims to provide students with an understanding that the direct addition operation of algebraic forms can be done with the same variables or the same type of menu price, but with different variables or types, it cannot be done. Based on Figure 6, students can understand that direct addition of algebraic forms cannot be done because they are not the same type or with the same variables (b). However, some students do not understand (a). Therefore, it is necessary to guide by adding one additional activity so that students are able to understand the meaning of the requested conclusion, namely that direct addition operations in algebraic forms can be carried out with the same variables or the same type of menu price, but cannot be carried out with different variables or types.



x is the price of iced tea, so the price of iced tea is not yet known for sure
y is the price of orange juice, so the price of orange juice is not yet known for sure

(a)



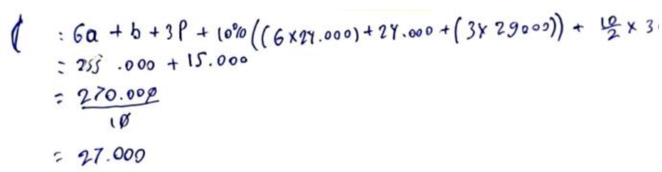
The conclusion is that the ones in the table above cannot be added because they are not the same.

(b)

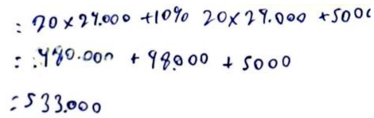
Figure 6. Example of Student Answers Performing Algebraic Operations

Using the properties of algebraic operations

This activity aims to enable students to use the properties of operations to simplify algebraic forms so that they can obtain the money paid or spent when having Iftar at a restaurant. Based on Figure 7, students use the properties of distributive algebraic operations (a), some do not use them (b), and some do not answer. Therefore, it is necessary to provide additional information by using brackets to group expressions in algebraic order to make it easier to see which algebraic operations are performed first and to see what properties of operations can be used to simplify algebraic forms.



(a)



(b)

Figure 7. Example of Student Answers Using the Properties of Algebraic Operations

Solving problems

This activity aims to enable students to solve the problem of money paid or spent when having Iftar at a restaurant. Based on Figure 7, some students can solve the problem of the situation in the context, some try to solve it but are not right (b) which makes the price of all menus the same as 24,000, some do not include tax costs in calculating the money that must be paid (a). Some students do not do this activity because they have not completed the previous activity, writing the algebraic form of a situation in the context with additional tax and parking costs. Therefore, guidance is needed in completing the activity of writing the algebraic form of a situation in a context with additional tax and parking costs, then carrying out algebraic operations and the properties of algebraic operations to obtain the amount of money paid or spent when Iftar at a restaurant, if the payment is divided equally.

Teaching experiment

Before the teaching experiment, the student activity sheet was revised according to the piloting results, including adding one activity number, adding additional information about taxes and how to calculate them, and adjusting the wording of the statement by adding parentheses to group expressions in algebraic order. Teaching experiments were conducted in class 7B MTs Laboratorium UIN Sulthan Thaha Saifuddin Jambi involving 18 students in two meetings of 2 x (2 x 40 minutes). Activity sheet 1 and activity sheet 2 were used in the first meeting, and activity sheet 3 was used in the second meeting. Students work collaboratively in groups of 3-4 students. Guided by the teacher, students state problems in algebraic form, perform algebraic operations, and solve problems based on their daily context, especially Iftar, as shown in Figure 1. Through the context of Iftar, students explore their experiences, state problems in algebraic form, identify algebraic forms, write algebraic forms, perform algebraic operations, use the properties of algebraic operations, and solve problems through the Indonesian Realistic Mathematics Learning Approach (PMRI), which emphasizes relevant contexts and student-centered learning.

- karna membayar Parkir
- Jumlah orang yg makan ada 3orang
- Pajak

- because of paying for parking
- the number of people eating is three
- tax

Figure 8. Example of Student Answers Identifying Things That Influence the Amount of Money Paid

Students are able to identify things that affect the amount of money paid, but they do not yet cover the whole thing. Groups 1, 2, 4, and 5 have not included the number of members and taxes that affect the amount of money paid, while group 3 has not included the food eaten during Iftar, and is still not right on the number of people eating, only three people as shown in Figure 8. Therefore, it is still necessary to be guided by observing and identifying information from the context given.

Researcher	:	*Do you understand the question: What are the factors that cause the amount of money to be paid when Iftar at a restaurant? What is the answer?*
Student	:	*Yes, and the answer is because of paying for parking. There are three people eating, and then there is the tax.*
Researcher	:	*There are three people. Is it true that there are three people?*
Student	:	*No.*
Researcher	:	*Actually, how many? How do we know how many people there are?*
Student	:	*Five people, known from the story or context.*
Researcher	:	*hen, are there other things that have an influence? Are these three enough?*
Student	:	*It seems enough.*
Researcher	:	*Think about it, if five people come, then have paid for parking, have paid tax, is there anything else that has not been paid?*
Student	:	*Eating.*
Researcher	:	*here is no one here yet? Why is that?*
Student	:	*Forgot.*

Based on observations and interviews, the guidance provided by observing and identifying information from the context provided helps students determine what causes much money to be paid, so that students can express a situation in algebraic form.

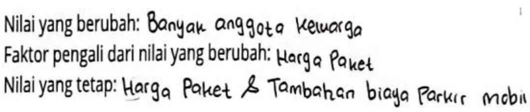
Banyak anggota keluarga	Harga Paket	Tambahan Biaya Pajak	Tambahan Biaya Parkir Mobil	Ekspresi Matematika/ Bentuk Aljabar
1	24.000	1 x 24.000 x 10%	5000	1 x 24.000 + (1 x 24.000 x 10%) + 5000
2	24.000	2 x 24.000 x 10%	5000	2 x 24.000 + (2 x 24.000 x 10%) + 5000
3	24.000	3 x 24.000 x 10%	5000	3 x 24.000 + (3 x 24.000 x 10%) + 5000
4	24.000	4x 24.000 x 10%	5000	4 x 24.000 + (4 x 24000 x 10%) + 5000
5	24.000	5x 24.000 x 10%	5000	5 x 24.000 + (5 x 24.000 x 10%) + 5000

Figure 9. Example of Student Answers Stating the Problem in Algebraic Form

All groups can state a situation in context with no additional costs and only one additional parking fee. Students still have difficulty stating a situation in the context of when there are two additional costs in a row, namely tax and parking fees, together in one algebraic form, as shown in Figure 9. They can state it by first understanding additional information about taxes and calculating tax costs in food shopping transactions.

Researcher : Do you understand what additional tax costs mean? The additional information sheet provided provides this information.
Student : Students read the additional information 2 again.
Researcher : So, how much is paid?
Student : The price of food plus the amount of tax.
Researcher : If we assume, based on the context of Iftar above, that there is one family member, how much does the food cost?
Student : 1 x 24.000
Researcher : How much is the additional tax cost?
Student : 10%
Researcher : From?
Student : 24.000, hmm..., 1 x 24.000
Researcher : If there are two of us, how much will the food cost?
Student : 2 x 24.000
Researcher : What does adding the tax mean?
Student : 2 x 24.000 plus 10% of 2 x 24.000
Researcher : If there are five people?
Student : 5 x 24.000 + 10% x 5 x 24.000
Researcher : So, how much do we have to pay?
Student : In addition to the food money, plus the tax.
Researcher : If so, what does the expression column contain?
Student : A combination of the number of family members, package price, additional tax, and additional car parking fees.
Researcher : So, if we want to pay for five people, how much?
Student : 5 x 24.000 + 10% of 5 x 24.000 + parking fee5000
Researcher : Can the information sheet help understand this problem?
Student : Yes.

Based on observations and interviews, the guidance provided by understanding additional information about taxes and calculating tax costs in food shopping transactions helps students express a situation involving two additional costs, namely tax costs and parking costs, together in one algebraic form using a table.



Changing value: Family members
Multiplier: Package prices
Constant value: Package prices & additional car parking fees

Figure 10. Example of Student Answers Identifying Algebraic Forms

Two out of five groups were able to identify the changing values, their multipliers, and the fixed values based on the algebraic form from the previous activity. However, the other three groups wrote the package price at a fixed value as shown in Figure 10, because they did not see the complete mathematical expression in the mathematical expression column, even though they had read the additional information sheet provided.

Researcher : Where do we get the information to determine the changing value, its multiplier, and the fixed value?
Student : In the column of the number of family members.
Researcher : Can we see the multiplier from that column?
Student : No.
Researcher : So, where can we easily get the information?
Student : The mathematical expression column.

Researcher : Where can we easily obtain information about changing values, multipliers, and fixed values?
Student : The mathematical expression column.
Researcher : So, what is the changing value?
Student : The number of family members.
Researcher : The multiplier of the changing value?
Student : Package price.
Researcher : Fixed value?
Student : Parking fee.

Based on observations and interviews, the guidance provided by observing and identifying information in the mathematical expression/algebraic form column from the previous activity and additional information 1 provided helps students identify changing values (variables), their multiplying factors (coefficients), and fixed values (constants).

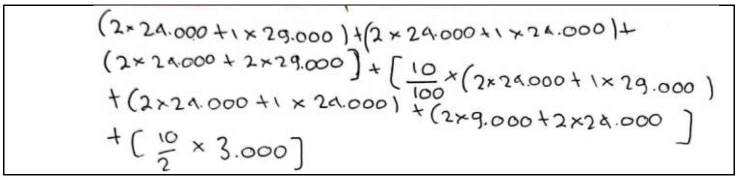


Figure 11. Example of Student Answers Writing Algebraic Forms

All groups were able to write the algebraic form of a situation in the context of no additional cost, with additional parking costs only, and with additional tax and parking costs. However, students still have difficulty writing the algebraic form of a situation in the context of additional tax and parking costs, consisting of many expressions as shown in Figure 11. There are also no students who have tried to write mathematical expressions in the form of variables to represent the prices of smashed chicken, grilled chicken, and catfish pecel.

Researcher : In order to get the amount of money needed, what do we do?
Student : Add up for each table, table 1 plus table 2 plus table 3.
Researcher : How much should be paid before tax?
Student : (2 x 24.000 + 1 x 29.000) table 1, plus (2 x 24.000 + 1 x 24.000) table 2, plus (2 x 24.000 + 2 x 29.000) table 3.
Researcher : What else do we have to pay in addition to this price?
Student : Tax, 10%.
Researcher : What does this mean?
Student : 10/100 multiplied by (2 x 24.000 + 1 x 29.000) plus (2 x 24.000 + 1 x 24.000) plus (2 x 24.000 + 2 x 29.000)
Researcher : So, how much do we have to pay if there is additional tax?
Student : The price to be paid before tax, plus 10/100 or 10% of the price to be paid earlier.
Researcher : So, is there anything else that needs to be added?
Student : Parking fee.
Researcher : Why is the parking 10/2 x 3.000?
Student : Because there are 10 people on one motorbike, consisting of 2 people.
Researcher : So, how is the expression?
Student : [(2 x 24.000 + 1 x 29.000) + (2 x 24.000 + 1 x 24.000) + (2 x 24.000 + 2 x 29.000)] + {10/100 x [(2 x 24.000 + 1 x 29.000) + (2 x 24.000 + 1 x 24.000) + (2 x 24.000 + 2 x 29.000)]} + (10/2 x 3000)

Based on observations and interviews, the guided use of parentheses to group expressions in algebraic order to make it easier to see which algebraic operations are performed first helps students write the algebraic form of a situation consisting of many expressions, such as additional tax and parking fees. Guidance is still needed in writing algebraic forms that contain variables that represent certain values, such as the price of smashed chicken, grilled chicken, catfish pecel, and parking fees.

Figure 12. Example of Student Answers Performing Algebraic Operations

All groups were able to understand the relationship between the algebraic forms on each table and the total algebraic form, namely that the total algebraic form is the sum of the algebraic forms of each table, but only two groups can conclude that direct addition can only be done with the same type of variable or the same type, the other three groups have not concluded it correctly as shown in Figure 12.

Researcher : *If we add the results in the total row, what is the result?*
Student : *3x + 7y.*
Researcher : *Why is the result 3x + 7y?*
Student : *Because the amount of iced tea is 13, and the amount of orange juice is 7.*
Researcher : *You add them, which are both iced tea alone and orange juice alone?*
Student : *Yes, you cannot add iced tea and orange juice directly.*
Researcher : *Why?*
Student : *Because there is a difference between iced tea and orange juice.*
Researcher : *So, what can we conclude?*
Student : *Direct addition can only be done with the same type of drink or the same variable.*

Based on observations and interviews, the guidance provided by concluding the relationship between the algebraic forms on each table and the total algebraic form helps students understand that direct addition operations of algebraic forms can be performed with the same variables or the same type of menu price, but cannot be performed with different variables or types. It should be emphasized that variable x indicates the price of iced tea and variable y indicates the price of orange juice, not the amount of iced tea and the amount of orange juice.

No group uses the distributive algebraic operation properties to simplify algebraic forms into simpler algebraic forms, as shown in Figure 13. Students tend to calculate directly so that they get the money paid or spent on Iftar at the restaurant. It happens because students are not yet able to see the overall structure of the expression, so that they can see which expressions can be simplified using the algebraic operation properties. It also occurs because the algebraic form in the previous activity does not contain variables to represent certain values, so that operations can be performed using the properties of algebraic operations.

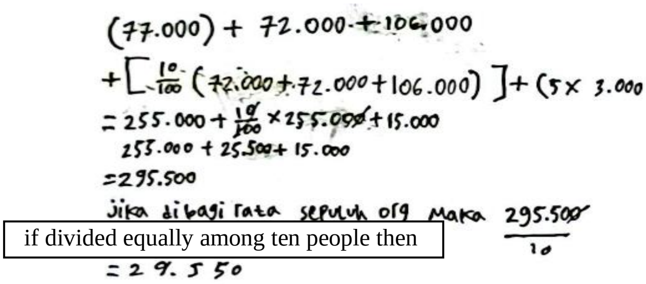


Figure 13. Example of Student Answers Using the Properties of Algebraic Operations

Researcher : *How do you get the results from the question: How much money does each person spend if the payment is divided equally?*
Student : *Adding the results from the previous activity.*
This, 77.000 + 72.000 + 106.000 + [$\frac{10}{100}$ x (77.000 + 72.000 + 106.000)] + (5 x 3.000)
Researcher : *Take a look at the previous activity. Do you see any package prices that are the same multiplied in the expression?*
Student : *Hmm..., it seems like there are.*
Researcher : *Which ones?*
Student : *2 x 24.000, 2 x 24.000, 1 x 24.000, 2 x 24.000 then there is 1 x 29.000, 2 x 29.000.*
Researcher : *Then what else?*

Student : *Added huh? Hmm., yes added, 2 x 24.000 + 2 x 24.000 + 1 x 24.000 + 2 x 24.000 + 1 x 29.000 + 2 x 29.000*
Researcher : *Try to pay attention to the same package price. How much is it?*
Student : *24.000*
Researcher : *What are the multipliers?*
Student : *2, 2, 1, 2*
Researcher : *Because the operation is addition, we can multiply 24.000 by (2 + 2 + 1 + 2), Try writing it down!!*
Student : *24.000 x (2 + 2 + 1 +2) = 24.000 x 7 = 168.000*
Researcher : *Try doing it at a package price of 29.000*
Student : *29.000 x (1 + 2) = 29.000 x 3 = 87.000*
Researcher : *What you just did was use the distributive property.*
Student : *Oh, yeah, yeah, I did not think of that before.*

Based on observations and interviews, the guidance provided additional information to use parentheses to group expressions in algebraic order to make it easier to see which algebraic operations are performed first, and see what properties of operations can be used in simplifying algebraic forms, which helps students simplify algebraic forms. However, students have not been able to use the properties of algebraic operations in the simplification process. Guidance is needed in writing algebraic forms that contain variables that represent certain values in previous activities, and see the overall structure of the expression so that one can see which expressions can be simplified using certain algebraic operation properties, whether commutative, associative, and distributive.

Four groups were able to complete activities 1, 2, and 3, which contained 17 activity numbers. Only one group did not complete the last activity (activity number 17) because they did not make optimal use of their time, so the learning time ended before activity number 17 was completed. The other four groups were able to solve the problem of money paid or spent during Iftar at a restaurant.

Conducting retrospective analysis

Retrospective analysis aims to analyze all data collected in learning experiments by comparing the HLT prepared and the Actual Learning Trajectory (ALT). The results of the retrospective analysis will be used to develop the HLT until it is considered sufficient to produce a better and more relevant LIT. The comparison of HLT and ALT for algebra learning in the context of iftar is shown in Table 3 below.

Table 3. Comparison of HLT and ALT of Learning Algebraic Forms with the Iftar Context

Activity	HLT	ALT
1. Students observe the menu of the Meriah Sambil Lahap package. 2. Students discuss the experience of iftar at a restaurant. 3. Students discuss what influences the amount paid for iftar at a restaurant. 4. Students try to state the problem in algebraic form using a table of component relationships that affect the money paid or spent when eating at a restaurant. 5. Students identify the changing values (variables), their	1. Students can find out the menu of the Meriah Sambil Lahap package. 2. Students share their experiences of iftar at a restaurant. 3. Students find that the number of family members who join, the food package price, additional tax costs, and parking affect the money paid. 4. Students can state the problem in algebraic form using a table (number of family members x package price + tax + parking). 5. Students can determine the values that change (number of family members and number of people who join), the multiplying factor (price of each package), and the fixed values (tax and parking).	1. Students know the menu of the Meriah Sambil Lahap package. 2. Together with group members, students share their experiences of Iftar at a restaurant. 3. Four groups found only the price of the food package and additional parking fees, and one group found only the number of family members who joined, as well as additional taxes and parking fees. Students have not found all the things that affect the money spent during Iftar at a restaurant. 4. Most groups can state the problem in algebraic form by first understanding additional information about taxes and calculating tax costs in food shopping transactions, and guided by the teacher. Students state the problem in algebraic form using the help of a table (number of

Activity	HLT	ALT
multiplying factors (coefficients), and the constant values (constants). 6. Students write the algebraic form of the money paid or spent for iftar at a restaurant.	6. Students write the algebraic form of the money paid or spent to eat at the restaurant: the number of family members x package price + tax + parking fee, and the Number of members x package price at each table + tax + parking fee according to the number of motorbikes.	family members x package price + tax + parking). 5. Two groups were able to identify the changing values, their multipliers, and the fixed values based on the algebraic form from the previous activity. However, the other three groups had not correctly identified the fixed values. They wrote the package price at a fixed value, which should have been at the multiplier. 6. All groups were able to write the algebraic form of a situation with no additional costs, with additional parking fees only, and with additional tax and parking fees. However, students still had difficulty writing the algebraic form of a situation with additional tax and parking fees, which consisted of many expressions. No students had tried to write mathematical expressions in the form of variables to represent the prices of smashed chicken, grilled chicken, and catfish pecel.
7. Students perform algebraic operations to obtain the money paid or spent for iftar at a restaurant. 8. Students use the properties of operations to simplify the algebraic form to obtain the money paid or spent when having iftar at a restaurant.	7. Students perform algebraic operations of addition, multiplication, and division to obtain the money paid or spent for iftar at the restaurant. 8. Students use the distributive property to simplify the algebraic form of money paid or spent when having iftar at a restaurant. Students do not use the distributive property to simplify the algebraic form of money paid or spent when having iftar at a restaurant.	7. Students were able to perform addition, multiplication, and division operations with the help of tables and direct calculations to obtain the money paid or spent during Iftar at a restaurant. All groups can understand the relationship between the algebraic form on each table and the total algebraic form, namely, the total algebraic form is the sum of the algebraic forms of each table, but only two groups can conclude that direct addition can only be done with the same type of variable or the same type. 8. All groups have not used the properties of distributive algebraic operations in simplifying algebraic forms into simpler algebraic forms. Students tend to calculate directly so that they get the money paid or spent during Iftar at the restaurant.
9. Students solve problems regarding money paid or spent when having iftar at a restaurant.	9. Students can solve problems regarding money paid or spent when having iftar at a restaurant, but some students cannot.	9. Four groups were able to complete activity 1, activity 2, and activity 3, which contained 17 activity numbers related to the problem of money paid or spent when having iftar at a restaurant. One group did not complete the last activity (activity number 17) because they did not make optimal use of time, so the learning time ended before activity number 17 was completed.

In this retrospective analysis phase, HLT is used as an initial framework to analyze students' responses to the learning process. Through the context of Iftar, students explore their experiences, state problems in algebraic form, identify algebraic forms, write algebraic forms, perform algebraic operations, use the properties of algebraic operations, and solve problems through the Indonesian Realistic Mathematics Learning (PMRI) approach. Students observe the package menu and discuss the experience of Iftar at a restaurant, then identify what influences the amount of money paid when having Iftar at a restaurant. Students can identify things that affect the amount of money paid, but they do not yet cover the whole thing. The guidance provided by observing and identifying information from the context helps students understand what causes much money to be paid, so that students can express a situation in algebraic form.

Students still have difficulty stating a situation in the context of two additional costs in a row, namely, tax and parking fees, together in one algebraic form. The guidance provided by understanding additional information about taxes and calculating tax costs in food shopping transactions helps students state a situation in an algebraic form involving two additional costs using a table. Two out of five groups can identify the changing values, their multipliers, and the fixed values, but the other three groups have not correctly identified the fixed values. The guidance provided by observing and identifying information in the mathematical expression/algebraic form column from the previous activity and additional information 1 provided helps students identify the changing values (variables), their multipliers (coefficients), and the fixed values (constants).

Students still have difficulty writing the algebraic form of a situation in the context of additional tax and parking costs, consisting of many mathematical expressions. No students have tried to write mathematical expressions in variable form to represent the prices of smashed chicken, grilled chicken, and catfish pecel. The guided use of parentheses to group expressions in algebraic order helps students write the algebraic form of a situation in a context consisting of many expressions. It is still necessary to guide in writing algebraic forms containing variables that represent certain values , such as the price of smashed chicken, grilled chicken, catfish pecel, and parking fees. It is necessary to modify the given guide by concluding the relationship between the algebraic form on each table and the total algebraic form to help students understand that direct addition operations of algebraic forms can be done with the same variables or the same type of menu price, but cannot be done with different variables or types. It is also necessary to pay attention to students' statements about variables, where variable x indicates the price of iced tea and variable y indicates the price of orange juice, not much iced tea and much orange juice.

Guided by providing additional information to use parentheses to group expressions in algebraic order to make it easier to see which algebraic operations are performed first, and see what properties of operations can be used in simplifying algebraic forms, helps students simplify algebraic forms. However, students are not yet able to use the properties of algebraic operations in the simplification process. Guiding is needed in writing algebraic forms containing variables that represent certain values in the previous activity, and seeing the overall structure of the expression, so that one can see which expressions can be simplified using certain algebraic operation properties, both commutative, associative, and distributive. Four groups were able to complete activity 1, activity 2, and activity 3, which contained 17 activities. Only one group did not complete the last activity (activity number 17) because it did not optimally utilize time, so the learning time ended before activity number 17 was completed. Context-based learning, such as Iftar, helps students understand concepts by providing relevant, guided experiences through guided reinvention to support them in reconstructing concepts and improving their problem-solving processes.

DISCUSSION

When designing learning activities, the key question is what meaningful problems can encourage students' cognitive development by the objectives of HLT. Three design principles guide the design process: Guided reinvention, didactical phenomenology, and emergent models (Doorman et al., 2016). In algebraic form, learning with the context of Iftar, the principle of guided reinvention is applied through contextual learning. Students solve problems of money paid or spent when having iftar at a restaurant, which encourages students to explore mathematical concepts in familiar situations. Pilot experiment and teaching experiment data show that guided reinvention helps students state problems in algebraic form, identify algebraic forms, write algebraic forms, perform algebraic operations, and solve problems of money paid or spent during Iftar at a restaurant more systematically. Some students have difficulty writing algebraic forms and solving problems of money paid according to students' cognitive processes when writing algebraic forms and solving problems, so they need different guidance (Kinanti et al., 2023).

Guided student activity sheets provide a framework to support students in developing a formal understanding of algebraic concepts (Mcuffey, 2018). Most students can write algebraic forms from the observed context (Agasi et al., 2017). Students are actively involved in the process of identifying and reinventing algebraic forms. This involvement is facilitated by cooperative group work, which helps students to better understand algebraic concepts (Flores & Park, 2016). In addition, the role of teachers in providing more interactive learning and motivating students to actively ask questions and discuss can also help reduce errors in completing student activity sheets (Saputra & Cesaria, 2023). In guided reinvention, teachers organize class discussions to help students develop mathematical ideas. This approach involves a significant shift of authority from teachers to students, encouraging them to take responsibility for their learning (Solomon et al., 2021).

Guided reinvention allows students to develop their understanding of logical relationships, which can be directly applied to understanding algebraic expressions and their logical structures (Dawkins & Cook, 2017). Students use several strategies to solve problems in context, including using tables and forming equations (Kwabena Tuffour, 2014). Using auxiliary tables allows students to visualize and manipulate algebraic expressions more effectively. It can lead to a deeper understanding of algebraic concepts and perform more accurate algebraic operations, making it easier for students to understand and solve problems (Bouck et al., 2025; Khairunnisak et al., 2021).

Didactic phenomenology is a meaningful context in students' lives through experience to encourage the development of mathematical objects that students learn (Doorman et al., 2016). The principle of didactical phenomenology is applied through the context of Iftar in three student activities. The first and third activities are given a menu list and problem situation, then students are asked to state the algebraic form and the amount of money paid. At the same time, the second activity is given a table of the ordered drink menu, and then students are asked to state the algebraic form. The context of Iftar is the starting point for students to state a situation in algebraic form and solve the problem of money paid or spent. Didactic phenomenology involves creating a hypothetical learning trajectory that guides students through the learning process systematically, ensuring they understand each step before moving on to more complex concepts (N. S. Utami et al., 2022). Through the developed HLT, students state a situation in the algebraic form of the phenomenon of Iftar, which is a progressive mathematization process. It involves students recognizing real-world phenomena and translating them into mathematical expressions through tables, then writing them in algebraic form. This process emphasizes starting with concrete experiences, such as money paid during Iftar, to build a foundation for understanding abstract concepts of algebraic forms (Rodríguez & Fernández, 2017). Through didactical phenomenology, students interact effectively and can express real-life problems as mathematical problems in the classroom (Ali, 2022).

Through the activity of writing algebraic forms and solving problems from the given context, a model that the students themselves construct emerges. The pilot experiment and teaching experiment data show that the algebraic mathematical model is written based on informal activities, such as observing package menus, discussing the experience of Iftar at a restaurant, and identifying what influences the amount of money paid during Iftar at a restaurant. The emerging model helps students reinvent formal mathematics by building on their informal understanding and contextual experiences (Gravemeijer, 1999). The use of models developed by students themselves serves as a bridge from real situations to abstract mathematical concepts. This process helps students develop their strategies and solutions (Pitriani, 2016). These models guide students through the process of abstraction, where they start with concrete and realistic problems and gradually develop more formal mathematical reasoning (Gravemeijer, 2020).

Several challenges are faced during the design and implementation process, including designing practical learning activities and anticipating diverse student thinking. Designing effective learning activities requires contextual relevance (Doorman et al., 2016) and an iterative design process (Gravemeijer & Cobb, 2006) so that learning is meaningful and relevant to students and perfect the designed learning activities so that they require in-depth exploration of the context and take a long time to perfect the learning trajectory. Anticipating diverse student thinking is done by understanding the diversity of thinking and predicting student learning trajectories that which requires flexible HLT (Gravemeijer & Cobb, 2006), making it challenging to design a trajectory that fits all.

CONCLUSION

This study successfully developed a Hypothetical Learning Trajectory (HLT) of algebraic form material and solving problems in the context of everyday life using algebra with the PMRI approach in the context of Iftar. Enhanced learning designs can promote students' understanding of algebraic forms and problem solving using algebra by integrating PMRI learning principles such as guided reinvention, didactical phenomenology, and emergent models. This principle contributes to the Local Instructional Theory (LIT), which emphasizes contextual relevance, collaboration, progressive mathematization, and systematic problem solving. This study also shows that integrating the context of Iftar provides a real experience that facilitates understanding of algebraic forms and problem-solving with algebra. However, challenges such as writing algebraic forms involving several situations related to other materials, such as tax costs and percentages, still arise.

Based on the findings of this study, it is recommended that teachers integrate relevant real-life contexts, such as iftar activities, into algebra learning to improve students' conceptual understanding and problem-solving abilities according to the principles of Pendidikan Matematika Realistik Indonesia (PMRI). For policymakers, the results of this study underscore the importance of supporting curriculum development and teacher training programs that promote the integration of cultural and social contexts in mathematics learning. It can explore students' relevant daily contexts to link algebra with other materials for further research.

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