Edumatica: Jurnal Pendidikan Matematika
Volume 15 Number 2 August 2025

Submited : 31 May 2025
Revised : 3 July 2025
Accepted : 2 August 2025

DOI: https://doi.org/10.22437/edumatica.vl5i2.44689

Students' Metacognitive Ability in Solving Mathematical Problems Based on Information Processing Theory in a Discrete Mathematics Course

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Abstract

This study aims to analyze students' metacognitive abilities in solving mathematical problems based on information processing theory in Discrete Mathematics courses. The study used a qualitative descriptive approach with a purposive sampling technique, involving two Mathematics Education students selected to represent the respondents' answer patterns. Data were collected through problem-solving tests using the think-aloud technique and unstructured interviews, then analyzed qualitatively with source triangulation. The results showed that both subjects were able to achieve the reflective use level on the first problem by going through all three stages of information processing. However, on the second problem, both only reached the strategy use level, with one subject experiencing pseudo in the thinking process. This finding confirms that students' metacognitive abilities vary depending on the complexity of the problem and the stages of information processing they go through. The implications of this study are the importance of developing learning strategies that encourage more consistent metacognitive regulation, as well as the opportunity to utilize technology such as eye-tracking or digital think-aloud tools to reveal students' thinking processes more objectively in future research.

Keywords: information processing theory; mathematical problem solving; metacognitive abilities

INTRODUCTION

Mathematical problem solving can help students build new mathematical knowledge. Problem-solving can stimulate interest in learning mathematics. Therefore, it can be said that mathematical problem-solving is an important area in mathematics teaching (Güner & Erbay, 2021a; Klang, Karlsson, Kilborn, Eriksson, & Karlberg, 2021). The problem-solving is the heart of mathematics (Majeed, Jawad, & Alrikabi, 2021; Polya, 2019; Yuniara, Saminan, Abidin, & Johar, 2023). According to him, a mathematical concept or principle only has meaning when it can be applied in solving everyday problems. Furthermore, Polya (2019) taught four stages of solving mathematical problems, namely: (1) Understanding the problem, namely students must identify the information that is known and what must be solved, (2) Planning how to solve it, namely students look for alternative answers that might be used, (3) Planning how to solve it, namely students follow the planned procedure and (4) looking back at the entire process that has been carried out, namely students evaluate the solutions found.

Metacognition plays an important role in supporting students in solving mathematical problems. Metacognition is students' awareness of the process and way of thinking about the things they do themselves, thereby improving the learning and memory processes (Braithwaite & Sprague, 2021; Flavell, 1979; Güner & Erbay, 2021a, 2021b; Gustiningsi, Putri, Zulkardi, & Hapizah, 2023; Gustiningsi et al., 2022; Yorulmaz, Uysal, & Çokçaliskan, 2021). Metacognition involves an understanding of the thinking process and the ability to regulate and control it.

In the context of mathematical problem solving, metacognition plays an important role, especially in improving learning and problem-solving abilities (Suliani, Juniati, & Lukito, 2022). According to (Utami, Setyosari, Fajarianto, Kamdi, & Ulfa, 2023) metacognitive abilities help students to: (1) Develop Learning Strategies, namely, students who understand metacognition tend to develop more effective learning strategies, (2). Monitor Understanding, namely, students can understand the extent to which they understand the material, and (3) Solve Problems, namely, the ability to plan and monitor problem solving is also improved through metacognitive understanding. Thus, metacognition plays a very important role in learning mathematics and in solving mathematical problems.

According toMagiera & Zawojewski,2011), there is a positive relationship between metacognitive activity and problem-solving implementation. The higher the level of metacognitive activity of students, the easier it is for them to solve mathematical problems. Furthermore, Magiera & Zawojewski (2011) stated that three metacognitive activities are relevant in solving mathematical problems, namely: (1) Metacognitive Awareness. This involves students' awareness of the thinking process and understanding of what will be done in solving the problem. For example, students can be aware of the steps that need to be taken to solve a mathematical problem. (2) Metacognitive Evaluation. This relates to students' assessments of the effectiveness of the chosen strategy in solving the problem. Students who have good metacognitive abilities can consider whether the strategy they are using is efficient or needs to be modified, and (3) Metacognitive Regulation: This involves students' ability to control their thinking during the problem-solving process. For example, students can change their approach if they feel the initial strategy is not working. By understanding and developing metacognition, students can enhance their mathematical problem-solving skills and acquire a broader knowledge base.

Based on preliminary data on the metacognitive abilities of undergraduate students of Mathematics Education in solving mathematical problems on Discrete Mathematics material registered in the 2023/2024 academic year, it was obtained: (1) Class R001 obtained the highest metacognitive ability score in problem solving of 17.5, the lowest score was 5.7 with an average score of 12.65, (2) Class R002 obtained the highest metacognitive ability score in problem solving of 19.7, the lowest score was 5 with an average score of 13.15, and (3) Class R003 obtained the highest metacognitive ability score in problem solving of 18.74, the lowest score was 0 with an average score of 10.42 with a maximum metacognitive ability score in problem solving of 40.

According to Swartz and Perkins (1989), the level of students' metacognitive ability in solving problems can be divided into four levels, namely: (1) Tacit Use: This is the use of thinking without awareness. Students at this level do not actively consider their thinking process when making decisions. They use thinking without thinking critically about the decision; (2) Aware Use: At this level, students begin to use metacognitive thinking with awareness. They consider their thinking process and understand how they solve problems. This awareness helps them choose more effective strategies; (3) Strategic Use: Students at this level use metacognitive thinking strategically. They select and organize problem-solving strategies wisely. They can also modify their approach if necessary, and (4) Reflective Use: This is the highest level of metacognitive ability. Students at this level reflectively consider their thinking process, identify strengths and weaknesses, and organize problem-solving very efficiently. Differences in individual cognitive styles can also affect students' ability to solve problems. By understanding and developing metacognitive skills, students can enhance their problem-solving abilities. There is a link between metacognitive abilities in solving mathematical problems and the information processing system. This theory explores how students process information through both short-term and long-term memory. According to Celikoz, Erisen, & Sahin (2019), information processing theory is defined as an effort by an individual to record, translate, store, and retrieve or recall information in their brain. According to Atkinson & Shiffrin (1968), there are three stages of information processing, namely: (1) sensory memory is information obtained from various series of stimuli through the senses such as visual, auditory and olfactory information, but most of it is ignored and not stored by the mind, (2) Short Term Memory is information that only lasts for 30 seconds and (3) Long Term Memory used to recall events in the past. Research on information processing theory has been conducted by Matitaputty, Mataheru, & Talib (2022), who stated that students already have an understanding related to permutations and combinations, but when determining problem-solving strategies, students forget the information in the problem at hand. Although several previous studies have examined mathematical problem solving using information processing theory (Nurhayati, Huda, & Suratno, 2020), these studies generally only emphasize how information is processed without specifically highlighting students' metacognitive dimensions. In fact, metacognitive abilities play a crucial role in regulating thought processes, selecting strategies, and reflecting on problem-solving steps taken. This research gap highlights the need to integrate information processing theory with metacognitive abilities to gain a more comprehensive understanding of how students actually think when confronting mathematical problems. Therefore, this study aims to explicitly analyze students' metacognitive skills in mathematical problem-solving based on information processing theory in Discrete Mathematics courses. This study is expected to provide theoretical contributions by enriching the literature on the relationship between metacognition and information processing, as well as practical contributions in designing more effective learning strategies to develop students' metacognitive regulation.

METHOD

This study employed a qualitative descriptive design with an exploratory orientation. The choice of this design was based on the objective of deeply analyzing students' metacognitive abilities in solving mathematical problems rather than testing hypotheses. A qualitative approach was considered appropriate because it allows for a detailed exploration of students' thought processes during problem solving.

The subjects of this study were two undergraduate students in Mathematics Education at Jambi University who had completed the Discrete Mathematics course. Purposive sampling was used to select students who were expected to provide rich information relevant to the research objectives. The process of selecting research subjects was carried out as follows: 1) Collecting students who would be prospective research subjects and determining the time for data collection for each prospective research subject; 2) Providing question sheets to prospective research subjects that were worked on with think-aloud; 3) Conducting interviews based on the question sheets worked on by prospective research subjects. This aims to obtain information about their thoughts that are not in think-aloud; 4) Grouping prospective subjects based on the saturated answers of prospective subjects. There are two groups of prospective subjects with saturated answers; 5) From each group, one subject was taken to be used as the research subject.

Research Instruments

The instruments used in this study consist of:

1. Main instrument. The primary instrument in this study is the researcher, who acts as both planner, data collector, data analyst, data interpreter, and reporter of research results.
2. Supporting instruments. The supporting instruments in this study were mathematical problem-solving sheets in the Discrete Mathematics course and interview lists. The question sheets used in this study were mathematical problem-solving questions designed to reveal students' metacognitive abilities in solving mathematical problems in the Discrete Mathematics course, and were administered through think-aloud protocols.

Data Analysis Techniques

The data analysis techniques used consist of:

1. Processing and preparing data for analysis. At this stage, the researcher transcribed the think-aloud data and interview results, and scanned the student assignment sheets. They typed the field data, sorted, and arranged the data into different types depending on the source of information.
2. Reading all the data by building a sense of the information obtained and reflecting on its meaning as a whole, such as the general ideas contained in the words of the research subjects. At this stage, the researcher took special notes or recorded general ideas about the data obtained.
3. Coding all data. Coding is the process of organizing data by collecting text sections or image sections. At this stage, researchers take written or image data that has been collected, segment sentences or images into categories, and then label the categories with specific terms.
4. Apply the coding process to describe the categories and themes to be analyzed. At this stage, researchers can create codes to represent all information and conduct analysis.
5. Present the results of the analysis in the form of a qualitative report. Researchers can use images or tables to enhance their presentations.
6. Make interpretations or give meaning to the data. Interpretation can be a meaning that comes from a comparison between research results and information from literature or theory. In this activity, researchers confirm whether their results confirm or deny previous information. Interpretation can also be a question that arises from data and analysis, which needs to be answered later
7. The data used is valid and reliable. The validity and reliability test used is source triangulation.

RESULTS

Research conducted with a given problem to the research subject, which is shown in Figure 1.

1. For any x and y, prove that the Boolean algebraic functions below are equal.

$$x + x'y = x + y$$

2. Prove:
   If $f(x_1, x_2, x_3) = (x_1 + x_2 + x_3)\ mod\ 2$, then the Boolean function will map $f\{0,1\}^3$ ke $f\{0,1\}$

Figure 1. The problem given

Solving the first question by the first Subject (S1)

The first subject completed the problem in 3 minutes and 33 seconds. At first, the subject read the problem and thought for a moment to understand the situation given. At the tacit metacognitive ability level, the subject reflected on whether this problem had been encountered before. Therefore, S1 recalled the mathematics material that had been studied in the discrete mathematics course. to determine how to solve the problem given. At this time, information enters through the sense of sight and is then stored for a while (sensory register recording stage). This is shown by the results of the researcher's interview with S1 as follows:

P : What did you think when reading question number 1?

S1: I think that the material given is related to ordinary algebra or Boolean algebra.

Furthermore, at the aware use level, the incoming information becomes S1's attention so that S1 is mindful of planning a solution to the problem given. S1 realizes that the problem presented is to prove that the Boolean algebra function on the left must be the same as the Boolean function on the right (planning a solution to the problem). This information enters short-term memory by being expressed repeatedly. This occurs at the short-term memory stage, which can be seen from the results of S1's work, as in Figure 2.

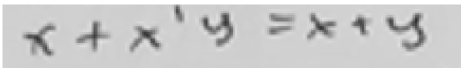


Figure 2. S1's answer when planning problem solving at the aware use level

Next, at the strategic use level, S1 thinks again and consciously S1 reveals a strategy that can be used to solve the equation x + x’y = x + y. According to S1, the strategy that can be used to solve x + x’y = x + y is to use the Boolean algebra law, namely the distributive law, the complement law and the identity law. This can be seen from S1's think-aloud, namely:

S1: To solve this problem, the Boolean algebra law can be used, namely the distributive law, the complement law, and the identity law.

Still at the strategic use level, S1 has the attention to use the distributive algebra law, the complement law and the identity law are forgotten, so S1 writes
(x + x’) + (x + y) = x + y (distributive law), 1 + (x’y) = x + y (complement law) and
x’y = x + y (identity law). S1 thinks again, then S1 crosses out the results he obtained. This can be seen in the following Figure 3.

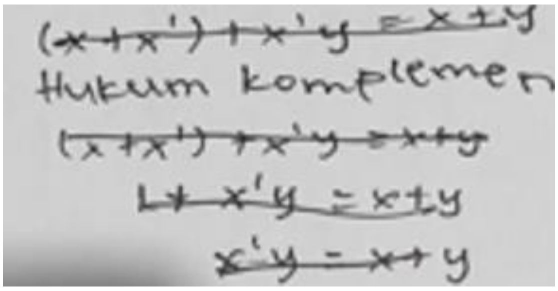


Figure 3. Results of S1's work when S1 made an error in elaboration

Next, S1 thinks back and elaborates on the stage of retrieving information from long-term memory about the distributive law, complement law and identity law in Boolean algebra, when using a plan in solving mathematical problems, so that S1 writes (x + x’) (x + y) = x + y (distributive law), 1 . (x + y) = x + y (complement law) and x + y =x + y (identity law). This can be seen from the results of S1's work in Figure 4 below.

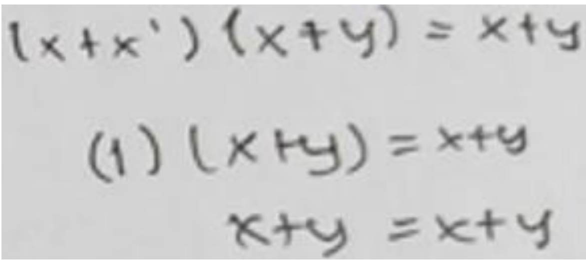


Figure 4. S1's work results during elaboration on long-term memory

S1's error in performing algebraic operations occurred due to interference in using the + and. signs in Boolean algebraic operations. The researcher's interview with S1 can demonstrate this.

P : What did you think back to when answering question no. 1

S1: Initially, I thought that the + sign was a sign for the largest lower limit and the. sign was a sign for the smallest upper limit.

To find out the level of metacognitive ability, S1 has reached the reflective use level, namely when checking back, S1 has reflected on his thinking process during the problem-solving process by considering improvements to his thinking results, namely thinking back that the plus sign (+) is used to determine the smallest upper limit and the dot (.) is used to determine the largest lower limit. This can be seen from the results of S1's think-aloud as follows.

S1: I remember again that the plus sign (+) is a sign of the smallest limit and the dot (.) indicates the largest lower limit.

Solving the second problem by the first subject (S1)

S1 solved problem number 2 for 2 minutes and 31 seconds. At the sensory register stage, S1 received information from the sense of sight. S1 read problem number 2 and thought for a while to understand the problem. S1 did not understand the problem as a whole. This can be seen from the results of the researcher's interview with S1 as follows.

P : Do you understand the problem as a whole?

S1: I read the problem as a whole, I understand that there is a function with three variables, namely x1, x2, and x3, which will be mapped to (x1 + x2 + x3) mod 2

S1 tries to solve the problem by making a truth table. At this time, S1's ability level is at the tacit use ability level, namely, S1 applies a method of solving problems through trial and error. This can be seen from the results of S1's think-aloud as follows.

S1: I tried to do it with a truth table

Furthermore, at the aware use level, S1 realizes that the problem-solving plan given gets attention from S1 by reading the problem repeatedly (short-term memory stage). Next, S1 solves the problem using the truth table. In this case, S1 writes down the possible truth values of x1, x2, x3, and the truth value of f(x1, x2, x3) as in Figure 5 below.

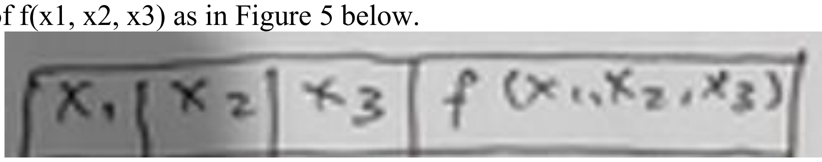


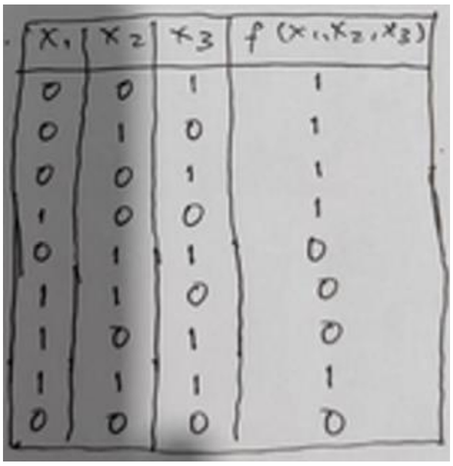
Figure 5. Results of S1 work in solving problems at the aware use level

At the level of strategic use ability, S1 has not been able to select a strategy that can be used to solve the given problem, but S1 can state that solving the problem can be done in one way, namely, using a truth table. This is evident from the interview results between the researcher and S1.

P : Do you have another way to solve the given problem?

S1: I don't think of another way to solve the problem, ma'am.

Next, S1 determines the truth value of x1, x2, and x3 as shown in Figure 6.



x_1	x_2	x_3	$f(x_1, x_2, x_3)$
0	0	1	1
0	1	0	1
0	0	1	1
1	0	0	1
0	1	1	0
1	1	0	0
1	0	1	0
1	1	1	1
0	0	0	0

Figure 6. Results of S1 work on solving problems using truth tables

S1 should have obtained eight possible truth values in his solution, but S1 got nine truth values. S1 did not recheck before, after, or during the problem-solving process. In other words, it can be said that S1's metacognitive ability did not reach the level of reflective use.

Solving the First Problem by the Second Subject (S2)

S2 completed the first problem in 3 minutes 41 seconds. First, S2 read the problem and thought for 31 seconds. Then, S2 read the problem again and immediately attempted to solve it. Therefore, it can be said that S2's ability level is at tacit use. When understanding a problem, information enters through the senses of sight and hearing, and is stored for a while. This occurs at the sensory register stages. At this stage, S2 will prove the Boolean function from left to right. This can be seen from the following think-aloud results.

S1: We try to prove this Boolean function from left to right, where x + x'y = x + y

Next, at the stage of planning problem solving at the aware use ability level, S2 will use the distributive law, the complement law, and the identity law.

At the aware use metacognitive ability level and when planning problem solving, S2 uses the distributive law, the complement law, and the identity law. This can be seen from the results of S2's work in Figure 7.

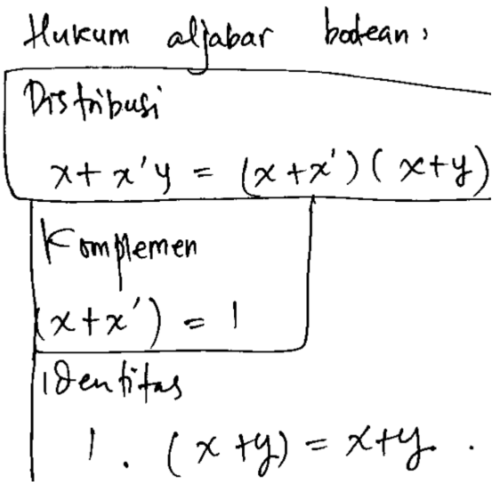


Figure 7. Results of S2 work on problem-solving plan at the ability level of Metacognitive aware use

At the level of strategic use of metacognitive ability, S2 realizes that to prove x + x'y = x + y, then S2 determines that S2 uses the distributive law, complement law, and identity law to implement the problem-solving plan given (short-term memory stage). This can be seen from the results of S2's work in Figure 8.

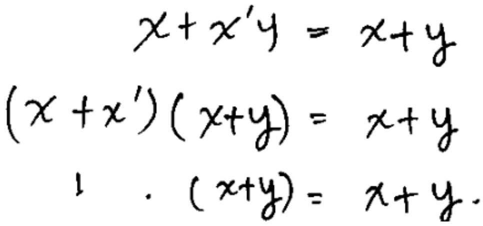


Figure 8. S2's work results when solving problems at the strategic use of metacognitive ability level

Next, at the reflective metacognitive level of use, S2 rechecks the results obtained from long-term memory, S2 rechecks the process before completion, at the time of completion, and at the end of completion. S2 can conclude that x = x'y = x + y. This can be seen from the results of S2's work in Figure 9 below.

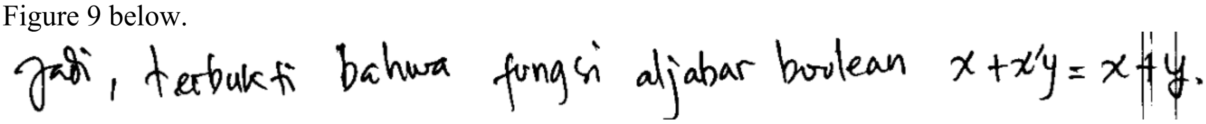


Figure 9. Results of S2's work at the metacognitive level of reflective use

Solving the Second Problem by the Second Subject (S2)

S2 completed the problem number in 5 minutes and 39 seconds. S2 read the tacit use problem number 2, and after reading the problem, S2 thought again while understanding the problem given. Information obtained from the sense of sight is stored in the sensory register at the sensory recording stage (sensory motoric). With hesitation (metacognitive tacit use level), S2 said that this problem can be solved using a truth table. This can be proven from the following think-aloud:

S2: This can be solved by using…….mmm using the truth table

At the short-term memory stage, S2 plans to solve the problem by drawing a truth table. S2 writes the variables x1, x2, x3 and x1 +x2 + x3 and f(x1, x2, x3) which can be seen in Figure 10.

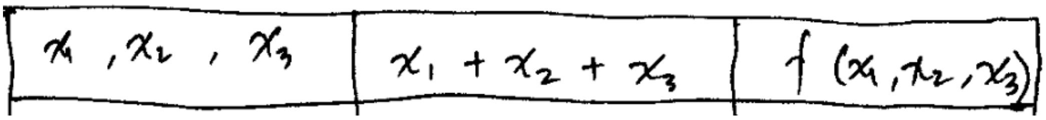
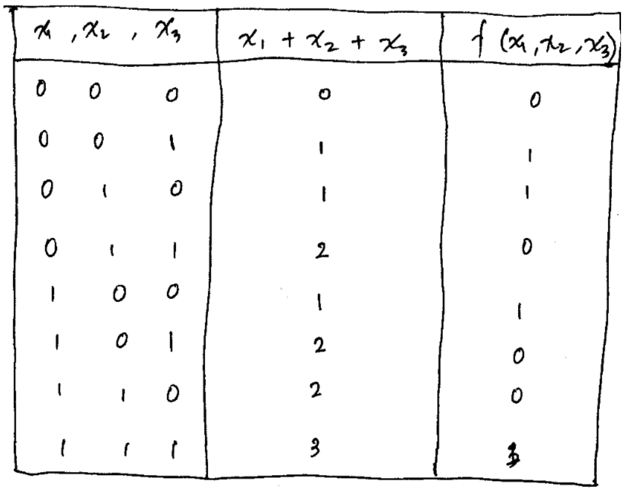


Figure 10. The results of S2's work illustrate the truth table at the metacognitive level aware use

Next, retention occurs in S2 at the short-term memory stage. S2 carries out the problem-solving plan by completing the truth table that S2 has created. At the strategic use ability level, S2 writes down all possible truth values of x1, x2, x3, the truth value of x1 + x2 + x3, and the truth value of f (x1, x2, x3). This can be seen in Figure 11.



x_1, x_2, x_3	$x_1 + x_2 + x_3$	$f(x_1, x_2, x_3)$
0 0 0	0	0
0 0 1	1	1
0 1 0	1	1
0 1 1	2	0
1 0 0	1	1
1 0 1	2	0
1 1 0	2	0
1 1 1	3	3

Figure 11. S2 work at the strategic use capability level when using a problem-solving plan.

Furthermore, during the recheck, S2 was unable to reach the level of reflective use of metacognitive ability. In other words, S2 did not reflect in his thinking process, so that Boolean algebra operations were carried out like ordinary algebraic operations. This can be seen from the following think-aloud results from S2:

S2 : 0,0,0 then if you add 0 then you get $(x_1 + x_2 + x_3)$ *mod2 is 0*
0,0,1 then if you add 1 then you get $(x_1 + x_2 + x_3)$ *mod2 is 1*
0,1.0 then if you add 1 then you get $(x_1 + x_2 + x_3)$ *mod2 is 1*
0,1,1 then if you add 2 then you get $(x_1 + x_2 + x_3)$ *mod2 is 0*
1,0,0 then if you add 1 then you get $(x_1 + x_2 + x_3)$ *mod2 is 1*
1,0,1 then if you add 2 then you get $(x_1 + x_2 + x_3)$ *mod2 is 0*
1,1,0 then if you add 2 then you get $(x_1 + x_2 + x_3)$ *mod2 is 0*
1,1,1, then if you add 3 then you get $(x_1 + x_2 + x_3)$ *mod2is 1*

During the calculation process using ordinary algebraic operations, a pseudo-occurrence occurs (the process is incorrect, but the results obtained are correct), namely, if $x_1 = 1, x_2 = 1, x_3 = 1$, then $(x_1 + x_2 + x_3)\ mod\ 2 = 1$ is obtained. This result is the same as the Boolean algebraic operation using the (+) sign as the most minor upper limit operation.

Comparison of S1 and S2

Both subjects achieved reflective use in the first problem, demonstrating the ability to monitor and regulate their thought processes. However, their performance declined in the second problem, where neither was able to reflectively evaluate their solutions. This contrast suggests that students' metacognitive abilities are sensitive to problem complexity and may fluctuate across tasks.

Table 1. Summarizes the metacognitive levels demonstrated by each subject in both problems.

Subject	Problem 1	Problem 2
S1	Reflective Use	Strategic Use
S2	Reflective Use	Strategic Use (pseudo)

DISCUSSION

Students' Metacognitive Abilities in Solving Boolean Algebra Problems

The findings of this study reveal that students exhibit varying levels of metacognitive ability when solving Boolean algebra problems. Both Subject 1 (S1) and Subject 2 (S2) demonstrated phases of thinking aligned with metacognitive dimensions—namely tacit use, aware use, strategic use, and, to some extent, reflective use. This finding aligns with the previous study, which emphasized that students who can regulate their learning strategies and reflect on their thinking processes tend to be more successful in mathematical problem-solving (Astriani, Susilo, Suwono, Lukiati, & Purnomo, 2020; Hastuti, Surahmat, Sutarto, & Dafik, 2020; Parwata, Jayanta, & Widiana, 2023; Rivas, Saiz, & Ossa, 2022).

In the first problem, both subjects successfully identified and applied Boolean algebra rules, such as the distributive, identity, and complement laws, indicating the use of structured cognitive strategies. S1, in particular, showed more advanced metacognitive capacity by recognizing and correcting symbol-related errors. This behavior demonstrates key metacognitive processes, including monitoring and self-regulation. Learners who can plan, monitor, and evaluate their learning tend to achieve better academic outcomes to (Matcha, Uzir, Gasevic, & Pardo, 2020; Wijaya, Zhou, Ware, & Hermita, 2021).

On the other hand, the initial error made by S1 and S2, due to their limited reflective capacity, may be related to cognitive load management. Fraser, Ayres, & Sweller (2015) highlighted that the effectiveness of problem-solving strategies is closely linked to students' understanding and management of their cognitive load. Students who fail to manage this load effectively may struggle to select appropriate strategy or perform accurate self-reflection.

Interestingly, both subjects demonstrated traits of a growth mindset, particularly in how they addressed mistakes and learned from their problem-solving experiences. Dweck (2008) stated that students with a growth mindset are more likely to take intellectual risks and view mistakes as opportunities for learning. S1's effort to revise their answer and S2's persistence despite uncertainty reflect their potential to further develop metacognitive thinking skills.

Weaknesses in Applying Truth Table Strategies

For the second problem, both subjects chose to construct a truth table as their problem-solving strategy. This suggests that students tend to rely on systematic approaches with which they are most familiar, even when they may lack a deep understanding of the conceptual framework. S1 made an error in generating nine combinations instead of eight for three binary variables, while S2 resorted to using conventional arithmetic instead of Boolean logic operations.

These inaccuracies indicate a lack of fully developed reflective use. One of the essential indicators of high metacognitive ability is the learner's capacity to evaluate both the process and outcome of their work independently and accurately. (Betul Kaya & Kepceoglu, 2022; Gustiningsi, Putri, Zulkardi, & Hapizah, 2024; Kartika & Muhassanah, 2023; Mansyur & Sunendar, 2020). The procedural errors also suggest that students may not yet be equipped to choose strategies appropriate to the complexity and goals of a given problem.

Pedagogical Implications

A comparison of the two subjects reveals that S1 demonstrated a stronger reflective capacity than S2. This supports the findings of Kholid & Ahadiyati (2022) who noted that students with higher metacognitive ability are better at identifying and correcting their own mistakes. As such, mathematics instruction at the tertiary level should be designed to systematically promote reflective thinking, rather than focusing solely on the correctness of final answers.

Practical strategies to foster metacognitive development include guided metacognitive prompts, scaffolded problem solving, and inquiry-based or project-based learning approaches. Moreover, it is crucial to expose students to a graduated series of cognitive challenges, allowing them to gradually learn to navigate and regulate their cognitive load in a conscious and structured manner.

Limitations and Future Research

This study is limited by its small sample size, which restricts generalizability. Data collection relied primarily on think-aloud protocols and interviews, which may not capture all aspects of students' cognition. Future studies could involve larger samples and incorporate technological tools such as eye-tracking and digital think-aloud systems to provide richer and more objective data. Such approaches would enhance the understanding of how metacognitive regulation interacts with information processing in mathematical problem-solving.

CONCLUSION

The metacognitive abilities of Subject One (S1) and Subject Two (S2) showed significant differences in their approaches to solving mathematical problems. S1 showed better development at all metacognitive levels. At the tacit use stage, S1 was able to relate the problem to previous knowledge. In aware use, S1 planned problem solving and identified relevant strategies. In strategic use, S1 demonstrated awareness in applying algebraic laws, although there were errors in their application. At the reflective use level, S1 reflected, which helped improve understanding, although it was not wholly effective in overcoming mistakes. Meanwhile, S2 showed limitations in the metacognitive process. Although starting from a tacit understanding, S2 did not fully grasp the problem. In aware use, although S2 was mindful of the need to use a truth table, limited attention and understanding hindered progress. In strategic use, S2 only applied one strategy without considering other alternatives. This demonstrated a lack of flexibility in S2, and S2 also failed to achieve reflective use, resulting in systematic errors in problem-solving. Recommendations for further research are: 1) to improve metacognitive learning, integrate learning strategies that focus on developing metacognitive skills, such as reflective thinking and modeling, into the mathematics curriculum. For example, guide students to actively reflect on their solution steps and understand the mistakes they make; 2) Provide exercises that encourage students to explore a variety of problem-solving strategies, not just one way. This can include group discussions and case studies that involve solving problems from multiple perspectives; 3) Provide constructive and specific feedback on students' thinking processes. Good feedback can help students understand their strengths and weaknesses in metacognitive thinking and more effective strategies to use. Recommendations for further research include conducting research using technology such as eye-tracking, learning analytics, or digital think-aloud tools to capture students' thinking processes in real-time and more objectively.

ACKNOWLEDGMENTS

This research was funded by the a postgraduate research grant from the Universitas Jambi, under research contract number 285/UN21.11/PT.01.05/SPK/2024.

DECLARATIONS

Author Contribution	:	Nizlel Huda: Conceptualization, Writing - Original Draft, Editing and Visualization; Syaiful: Validation and Supervision; Tria Gustiningsi: Writing -Review & Editing Novferma: validation and writing.
Funding Statement	:	This research was funded by the a postgraduate research grant from the Universitas Jambi, under research contract number 285/UN21.11/PT.01.05/SPK/2024 for supporting and funding this research.
Conflict of Interest	:	The authors declare no conflict of interest.
Additional Information	:	Additional information is available for this paper.

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