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Spatial Literacy and Error Patterns in Elementary 3D Geometry: Foundations for Local Instructional Theory

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Abstract

Spatial literacy is a vital math skill for students to learn during their three-dimensional geometry education. The two skills remain essential, yet most fifth-grade students struggle with spatial tasks. This challenge restricts their conceptual growth. The research examines fifth-grade students' spatial literacy abilities and their error patterns to provide evidence for developing a Local Instructional Theory (LIT) on three-dimensional geometry. The research included 118 fifth-grade students (aged 10-11 years) from two public primary schools in Palembang who participated in a descriptive quantitative study with error analysis. The research data came from student performance on the spatial literacy diagnostic test and their error analysis records. The researchers applied descriptive statistics to determine average scores and establish performance levels for visualisation, reasoning, and communication skills. The researchers categorised student mistakes into four categories: procedural errors, conceptual errors, representational errors and communicative errors. The students produced results indicating significant differences in their performances. Students failed to perform basic procedural tasks, including spatial form comparison and application of formal notation, while they struggled with complex visualisation and the development of multiple-step spatial thinking and conceptual understanding. Students' learning difficulties manifest as systematic errors, indicating that their challenges stem from complex, interconnected issues rather than random mistakes. The research results demonstrate that students need instruction which teaches them to develop their visualisation abilities, their reasoning skills, and explanatory competencies. The designs serve as an essential tool for uniting procedural fluency with conceptual understanding, leading to better spatial literacy in early geometry education.

Keywords: elementary students; error patterns; local instructional theory; spatial literacy; three-dimensional geometry

INTRODUCTION

Elementary school students need to learn about three-dimensional (3D) geometry through cubes and cuboids to develop their spatial literacy skills. Spatial literacy requires students to develop three essential skills, which include reasoning, visualisation, and spatial communication, for understanding object relationships in concrete and abstract settings. Research shows that this ability functions as a key factor which determines student achievement in science and technology, mathematics, and visual arts subjects (Lai, Mustafa, & Mahat, 2024; Sutçu, 2021; Lowrie, Logan, & Ramful, 2017).

The national learning objectives of the Indonesian elementary math curriculum, Kurikulum Merdeka, include spatial literacy as an essential component. Students in the Geometry and Measurement curriculum need to develop spatial vision skills and learn to identify geometric relationships and understand shapes through different representation methods (Kemdikbudristek, 2024; Saa, 2024). The curriculum focuses on meaningful, culturally relevant education through its main objectives, which include mathematical reasoning and contextual problem-solving. The focus on spatial visualisation in education matches the requirements of international standards, including the PISA Mathematics Framework 2022, which identifies Shape and Space as an essential subject for developing mathematical literacy and spatial reasoning abilities (OECD, 2023). The Common Core State Standards and Programme for International Student Assessment (PISA) include geometry problem-solving and reasoning, and spatial visualisation as essential skills which should be developed in students from an early age (Association, 2020). Students need to demonstrate strong spatial literacy skills to connect geometric concepts with actual events in their everyday lives.

Research findings indicate that students face ongoing difficulties in understanding three-dimensional geometry. Several studies show that students have trouble recognising geometric nets, distinguishing between faces, edges, and vertices, and correctly figuring out surface areas and volumes (Pujiastuti, Hidayat, Hendrayana, & Haryadi, 2024; Katuuk & Tarusu, 2024; Gal & Linchevski, 2010; Pittalis & Christon, 2010). The combination of these problems reveals consistent errors, which demonstrate that students face challenges in understanding solid shapes because of their conceptual and operational weaknesses (O'Rear et al., 2023; Rodríguez, Salsa, & Martí, 2021; Fujita et al., 2022).

The assessment results from national and international organisations confirm that students face multiple learning difficulties. The 2023 Asesmen Kompetensi Minimum (AKM) results in Indonesia revealed that primary school students performed poorly in math reasoning and geometry tests, which required them to depict and measure space (Kemdikdasmen, 2024). The PISA 2022 assessment of Indonesia revealed that only 33% of students could apply geometric reasoning to solve real-world problems, which was significantly lower than the OECD average (OECD, 2023). The research findings demonstrate an immediate requirement for new teaching methods which should develop spatial literacy skills from the beginning of education.

The research findings demonstrate that current teaching approaches fail to develop. The 21st century mathematics education requires spatial literacy as a fundamental skill, according to international research (Lane, Lynch, & McGarr, 2019). Research from recent years demonstrates that students' problem-solving abilities in science and technology fields heavily depend on their spatial abilities ((Gagnier, Holochwost, & Fisher, 2022; Ramulumo, 2024). The integration of spatial skills into elementary education faces challenges because both curriculum development and teacher pedagogy lack sufficient support (Suer & Demirkol, 2023).

Most research about spatial abilities and geometric reasoning has focused on high school and college students (Turgut & Yılmaz, 2012; Azzahra, 2022; Sutarna & Maryani, 2021). Research about elementary students' spatial literacy development in relation to their 3D geometry problem-solving errors remains scarce (Fiantika, Maknun, Budayasa, & Lukito, 2018).

The research investigates students at the elementary school level because this stage enables students to develop their fundamental spatial reasoning abilities. The error analysis method enables researchers to understand student misconceptions and cognitive structures, which form the basis for creating an effective Local Instructional Theory (LIT) that guides teaching methods. The research examines students' spatial literacy development through their mental operations by analysing common mistakes to inform improved teaching approaches for specific learning challenges.

The research distinguishes itself through its dual methodology: it assesses elementary students' spatial literacy in 3D geometry through cube and cuboid evaluation, and establishes a Local Instructional Theory (LIT) framework using local cultural artefacts and PMRI principles. The combination of spatial literacy analysis with local context and instructional theory development remains an underexplored area in previous research.

Research has established that children face difficulties with 3D geometry, but no study has established direct links between these difficulties and spatial literacy profiles through primary school error analysis. The research provides essential diagnostic information and educational strategies that align with national curriculum requirements and international literacy standards.

METHOD

The research employed descriptive quantitative methods with error analysis for data collection. The research used descriptive methods to identify patterns in students' spatial literacy, but error analysis revealed systematic problems students exhibited when solving 3D geometry tasks involving cubes and cuboids. The researchers selected this method to achieve two goals: to show score distributions and to identify student learning difficulties. This collected data serves to develop a Local Instructional Theory (LIT).

Participants

The research involved 118 fifth-grade students aged 10-11 from two public primary schools in Palembang, Indonesia. The researcher selected schools using purposeful sampling to achieve a diverse range of student backgrounds and school environments. The selection process focused on schools operating under the Kurikulum Merdeka and willing to conduct spatial literacy assessments. The principals of the schools and math teachers provided their approval before data collection started, and parents gave their consent for their children to participate. The researcher selected entire classes for data collection rather than randomly selecting students to maintain authentic classroom environments, which aligned with the research goal of studying real learning processes. All students from the selected classes participated voluntarily in the research.

Instruments

The research used two main assessment tools, which included a spatial literacy assessment based on 10 indicators that measured visualisation reasoning, and communication abilities (Octaria & Rawani, 2025). The test consisted of two sections, including eight open-ended cube and cuboid questions to assess students' 3D shape visualisation and understanding, problem-solving abilities, and an error-analysis sheet to organise incorrect responses into three categories: net-interpretation errors, surface-area calculation mistakes, and face-edge-vertex confusion.

Validity and Reliability

Three math education professors evaluated the content validity of the test items through their expert judgment. The experts evaluated each test item to verify its relevance to the topic, its clear language, and its effectiveness in demonstrating spatial literacy indicators. The researchers incorporated expert feedback to modify the test items before conducting the pilot test. The product-moment correlation method was used to evaluate empirical validity through data from 30 Grade 5 students who were not part of the main research group. The test items achieved validity when their calculated correlation coefficient exceeded the table value of $r_{value} > r_{critic} = 0.361(\alpha = 0.05)$. The reliability assessment used Cronbach's Alpha to determine if $\alpha \geq 0.70$ for acceptance. The researcher conducted reliability analysis through SPSS 25.0. The researcher eliminated or modified test items which demonstrated negative item-total correlations and weak discrimination power. The instrument development process included evaluation of item difficulty and discrimination indices during pilot testing to improve the instrument. The expert team verified the completed test to confirm it met the content and empirical standards for primary school geometry assessment.

Data Collection

The data collection process followed a structured method, which included (1) instrument development, (2) expert validation, (3) pilot testing and revision, (4) distribution to the primary participants, (5) digitalisation and coding of the data, and (6) error analysis to identify student learning obstacles. The teacher monitored students as they completed all assessment tasks through paper-and-pencil activities during their standard classroom hours.

Data Analysis

The researchers applied a 0-3 scoring system to assess student responses, where 0 indicated incorrect or no answer, 1 indicated partial correctness, 2 indicated reasonable correctness, and 3 indicated complete correctness. The researcher applied range score classification to determine average scores for each indicator, which received three categories of low, medium, or high according to (Azwar, 2012; Widoyoko, 2012; Arikunto, 2021). The quantitative analysis improved through qualitative error-type coding, which revealed students' persistent misconceptions and logical processing errors. The researchers identified the most common error patterns in the spatial literacy domains by calculating error frequency and percentage for each category. The researcher used SPSS 25.0 to produce descriptive statistics, including means, standard deviations, percentages, and cross-tabulations, to understand students' performance and error patterns in depth. The combination of descriptive and error analyses provided a detailed understanding of students' spatial literacy and revealed critical needs for classroom intervention.

RESULTS

The spatial literacy test tool underwent validation through two distinct stages of evaluation. The first evaluation phase assessed content validity through expert evaluation by three math education professors who specialised in the field. The experts verified that all tasks measured spatial literacy skills through visualisation and reasoning, and communication, and they confirmed the language was suitable for Grade 6 students. The team made small changes to the tool after receiving feedback from experts. The tool underwent empirical validity testing with 30 Grade 6 students from schools that did not participate in the main research study. The product-moment correlation analysis generated item validity coefficients ranging from $r_{value} = 0.376$ to $r_{value} = 0.745$, which surpassed the minimum requirement of $r_{critic} = 0.361$. The assessment proved that all test items met the necessary standards for test usage.

The instrument's reliability was measured through Cronbach's Alpha coefficient assessment. The results showed $\alpha = 0.730$, which surpassed the minimum requirement of 0.70. The test results showed that the instrument achieved appropriate internal consistency and reliability for assessing spatial literacy skills in primary school students.

The descriptive analysis revealed that students demonstrated different levels of competence when working with 3D geometry tasks. The 118 students who participated in the study achieved an average score of 1.23, with a standard deviation of 1.01. The students achieved the lowest score of 0 and the highest score of 3. The student distribution across categories revealed that 24.57% students belonged to the low group, while 75.43% students fell into the middle group, and no students reached the high group.

The results showed performance levels varied across areas. Students achieved their highest score in communication, with a mean of 1.40, followed by logic ($\bar{x} = 1.21$). The visualisation domain received the lowest score with a mean of 1.03. These results show that students demonstrated better ability to explain spatial concepts, but their spatial visualisation skills remained weak.

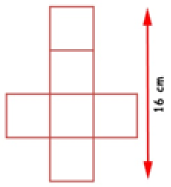
Table 1. Descriptive Statistics of Students' Spatial Literacy

Spatial Literacy Domain	Indicator	Max Score	Mean	SD	Category
Visualization	Students need to draw spatial objects by creating visual sketches which follow the instructions from the problem	3	1.26	0.89	Medium
	Students need to create visual aids which help them solve problems	3	1.28	0.94	Medium
	The process involves changing a depicted object into a new shape	3	0.55	0.66	Low
Reasoning	Students need to study how spatial objects relate to each other	3	0.56	0.67	Low
	Students need to evaluate the connections between spatial objects	3	2.46	0.67	High
	Students need to handle the relationships between spatial objects	3	0.60	0.64	Low
Communication	Students need to locate all relevant details which appear in the problem	3	1.28	0.80	Medium
	Students need to share their thoughts about objects and their spatial connections through spoken or written communication	3	1.28	0.80	Medium
	The correct application of mathematical terminology and symbols and notation systems for spatial objects and their relationships	3	2.45	0.63	High
	Problem-solving activities produce accurate results, which lead to correct conclusions	3	0.60	0.64	Low

The analysis results in Table 2 show different levels of spatial literacy performance among students across the three domains, which include ten indicators. The two visualisation domain indicators about sketching spatial objects and creating visual aids for problem-solving received medium ratings ($\bar{x} = 1.26, SD = 0.89\ and\ \bar{x} = 1.28, SD = 0.94$). The students demonstrated an average ability to link visual elements to problem contexts, but their performance showed significant individual differences due to the high standard deviations. The students performed poorly on the task of changing a represented object into a new form because they received a low rating ($\bar{x} = 0.55, SD = 0.66$). The students faced difficulties with visual transformation tasks because they required advanced spatial thinking. In other words, when asked to re-represent an object in a different form, many students failed to maintain the spatial structure of the original object.

An example of an item for the indicator "drawing spatial objects through visual sketches based on a given problem" is shown in Figure 1. Student responses revealed a wide range of strategies as well as errors in interpreting visual representations

Ari wants to make a large dice out of paper for playing. He draws a net as shown in th following figure.

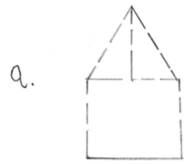
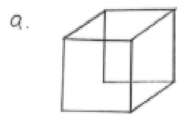
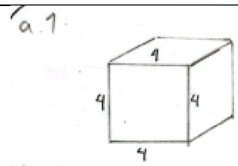


a. Draw the three-dimensional solid that will be formed from the net after it is folded!

Figure 1. Item No. 4a

Analysis of students' responses to this indicator (see Table 2) highlights significant visualisation errors. A total of 62.7% of students either left the item blank, drew an unrelated shape, or produced only a net. Another 28.8% made representational errors, such as drawing a cube that appeared distorted into a cuboid or lacked proportionality. Only 8.5% of students' initial ability to read and generate spatial visual representations remains limited.

Table 2. Student Errors on Indicator 1a

Proficiency Category	Student Errors	Student Responses
Low	Visualisation errors (no response, drawing other shapes, drawing only a net)	
Medium	Representational errors (drawing a cube resembling a cuboid, lack of precision)	
High	None	

An example item for the indicator, "developing visual representations to support problem-solving," is shown in Figure 2. Students' responses illustrate a wide range of strategies and errors in interpreting conceptual visual representations.

A cuboid has a length of 6 cm, a width of 4 cm, and a height of 3 cm.

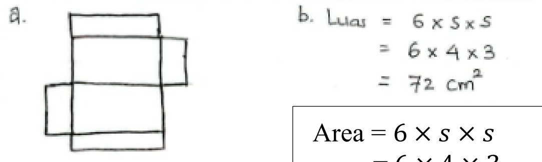
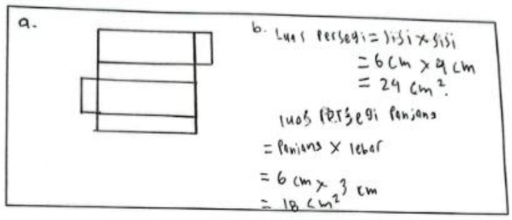
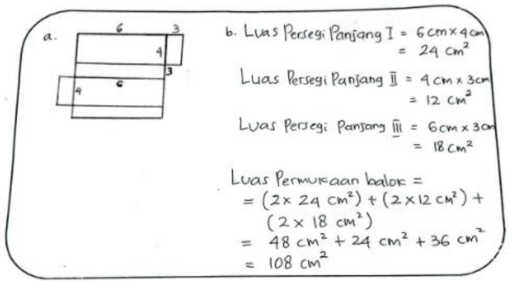
a. Draw the net of the cuboid.

b. Use the net to help you calculate the surface area of the cuboid.

Figure 2. Item No. 6

For this indicator (Figure 2; Table 3), students' errors were more diverse. A total of 60.2% of students exhibited visualisation errors (e.g., no response or inability to correctly draw a cuboid net), conceptual errors (e.g., misunderstanding that the surface area of a cuboid is the sum of all its faces), and procedural errors (e.g., replacing surface area with volume). In addition, 28.8% of students demonstrated representational errors, such as drawing a cuboid net with incorrect side proportions, and procedural errors, in which only partial steps in calculating the surface area were correct, leading to an incorrect final result. Only 11% of students solved the problem successfully.

Table 3. Student Errors on Indicator 1b

Proficiency Category	Student Errors	Student Responses
Low	Visualisation errors (no response, incorrect cuboid net), conceptual errors (failure to understand that surface area is the sum of all faces), procedural errors (substituting volume for surface area)	 Area = $6 \times s \times s$ $= 6 \times 4 \times 3$ $= 72\ cm^2$
Medium	Representational errors (cuboid net with inaccurate proportions), procedural errors (partially correct surface area calculation but incorrect final result)	 The area of a square = $side \times$ side $= 6\ cm \times 4\ cm$ $= 24\ cm^2$ The area of a rectangle = $length \times width$ $= 6\ cm \times 3\ cm$ $= 18\ cm^2$
High	None	 The area of rectangle I = $6\ cm \times 4\ cm$ $= 24\ cm^2$ The area of rectangle II = $4\ cm \times 3\ cm$ $= 12\ cm^2$ The area of rectangle III = $6\ cm \times 3\ cm$ $= 18\ cm^2$ The surface area of the cuboid $= (2 \times 24\ cm^2) + (2 \times 12\ cm^2) + (2 \times 18\ cm^2)$ $= 48\ cm^2 + 24\ cm^2 + 36\ cm^2$ $= 108\ cm^2$

An example item for the indicator transforming a represented object into a different form is presented in Figure 3. Students' answers revealed varied strategies and difficulties in understanding spatial transformation.

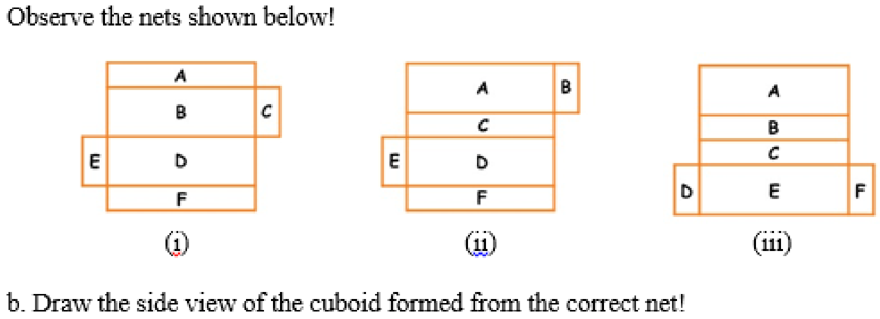
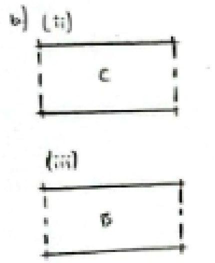
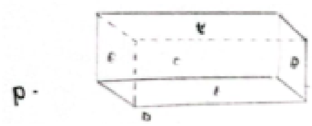


Figure 3. Item No. 5b

For this indicator (Figure 3; Table 4), student achievement was the lowest across all visualisation tasks. A total of 90.68% of students made visualisation errors, such as leaving the item blank or failing to draw the side view of a cuboid. The remaining 9.32% of participants made representational errors through their side-view drawings, which showed partial accuracy but contained wrong measurements. Notably, no students provided a completely correct answer for this indicator.

Table 4. Student Errors in Indicator 1c

Proficiency Category	Student Errors	Student Responses
Low	Visualisation errors (no response, failure to draw the side view of a cuboid)	
Medium	Representational errors (side view partially correct, but with inaccurate proportions)	

In the reasoning domain, the analysis showed that the indicators of analysing concepts and relationships of spatial objects ($\bar{x} = 0.56, SD = 0.67$) and managing concepts and relationships of spatial objects ($\bar{x} = 0.60, SD = 0.64$) fell into the low category. In contrast, the indicator of comparing concepts and relationships of spatial objects ($\bar{x} = 2.46, SD = 0.67$) was classified as high. These differences indicate variations in difficulty across indicators within the reasoning domain.

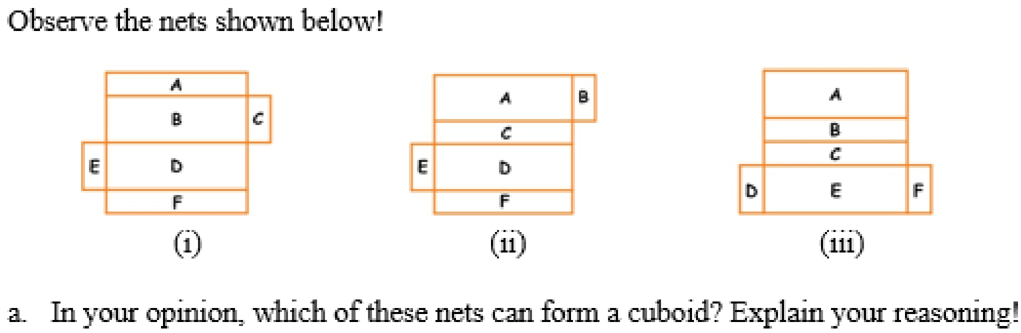
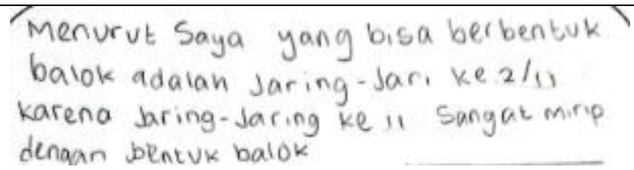
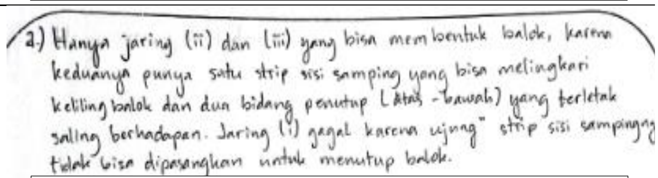


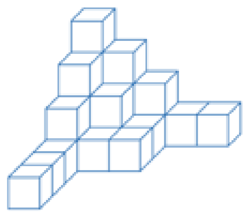
Figure 4. Item No. 5a

The item for indicator 2a (analysing concepts and relationships of spatial objects) is presented in Figure 4, and student responses are summarised in Table 5. A total of 89.8% of students made visualisation errors (e.g., leaving the item unanswered, assuming all nets could form a cuboid, or claiming none of the nets were correct) and conceptual errors (e.g., selecting a net randomly without justification or providing incorrect reasoning). Meanwhile, 10.2% of students demonstrated representational errors, such as correctly identifying some nets but misidentifying others, often with incomplete reasoning. Notably, no students solved the item correctly.

Table 5. Student Errors in Indicator 2a

Proficiency Category	Student Errors	Student Responses
Low	Visualisation errors (no response, all nets considered correct, no net considered correct); conceptual errors (random answers without justification or with incorrect reasoning)	 In my view, the second net (ii) can form a cuboid, as it closely resembles the structure of a cuboid
Medium	Representational errors (partially correct identification but incorrect for other nets); partial conceptual errors (incomplete reasoning)	 Only nets (ii) and (iii) can form a cuboid, as both contain a strip of lateral faces that can wrap around the cuboid's perimeter and two covering faces (top and bottom) that are positioned opposite each other. Net (i) fails because the ends of its lateral strip cannot be joined to enclose the cuboid

Observe the following solid composed of unit cubes!

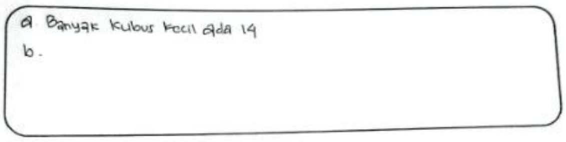
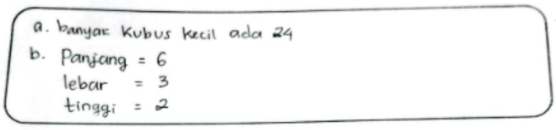


a. Calculate how many small cubes make up the solid.
b. If all the small cubes are rearranged into a single cuboid, determine its length, width, and height.

Figure 5. Item No. 7

For indicator 2c (managing concepts and relationships of spatial objects), the corresponding item is shown in Figure 5, and the student responses are presented in Table 6. Results indicate that 91.5% of students committed conceptual errors, such as miscounting the number of cubes, guessing without justification, or leaving the item unanswered. Another 8.5% of students made procedural errors, such as partially correct calculations of cube quantities but incorrect determination of the cuboid's length, width, and height. None of the students provided a fully correct response.

Table 6. Student Errors in Indicator 2c

Proficiency Category	Student Errors	Student Responses
Low	Visualisation errors (no response, failure to account for hidden layers); conceptual errors (incorrect determination of cuboid dimensions)	 a. The total number of small cubes is 14 b.
Medium	Representational errors (omitting back columns or bottom layers); partial errors in determining cuboid dimensions	 a. The total number of small cubes is 24 b. Length = 6, width = 3, height = 2

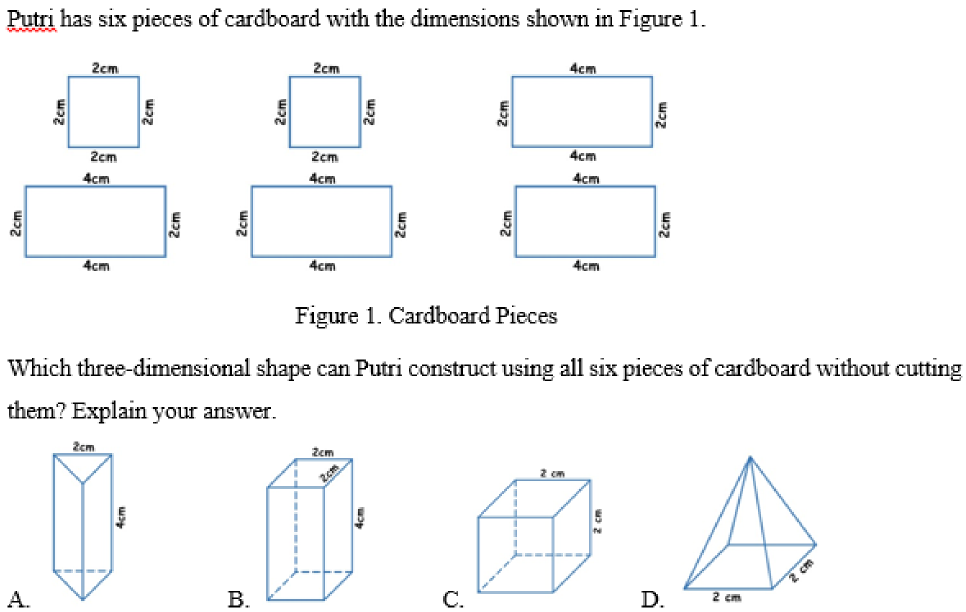
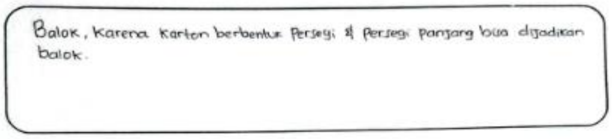
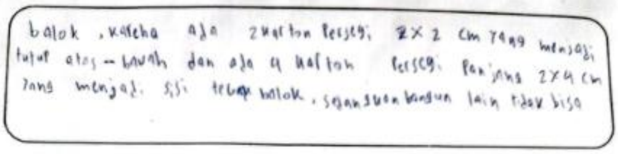


Figure 6. Item No. 1

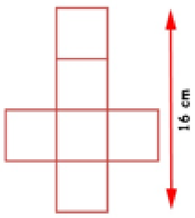
For indicator 2b (comparing concepts and relationships of spatial objects), the relevant item is shown in Figure 6, and student responses are summarised in Table 7. Results showed that 11% of students made visualisation errors (e.g., no response, copying the net without identifying the 3D solid) and conceptual errors (e.g., selecting an incorrect solid or naming a cube without justification). Additionally, 33.9% of students exhibited representational errors, such as correctly identifying the solid but failing to explain the correspondence of faces in detail, or partial conceptual errors (e.g., naming a cube even though the side lengths differed). A total of 55.1% of students successfully solved the item correctly.

Table 7. Student Errors in Indicator 2b

Proficiency Category	Student Errors	Student Responses
Low	Visualisation errors (no response, copying the net without identifying the solid); conceptual errors (incorrect solid, naming a cube without justification)	 B. a cube, because all of its dimensions are equal
Medium	Representational errors (correct answer but insufficient explanation of face correspondence); partial conceptual errors (naming a cube despite unequal side lengths)	 A cuboid, because the cardboard pieces in the form of squares and rectangles can be assembled into a cuboid
High	None	 A cuboid, because there are two square cardboard pieces measuring 2 x 2 cm that serve as the top and bottom covers, and four rectangular cardboard pieces measuring 2 x 4 cm that form the vertical faces of the cuboid, whereas the other shapes cannot be constructed

In the communication domain, the indicators identifying given information in the problem ($\bar{x} = 1.28, SD = 0.80$) and expressing ideas related to spatial objects or relationships accurately in written or oral form ($\bar{x} = 1.28, SD = 0.80$) were classified as medium. These results suggest that students were able to communicate partial spatial information, although the skills demonstrated remained fragmented and inconsistent. The indicator, which employed mathematical symbols, spatial object notation, and relationship symbols, achieved high accuracy ($\bar{x} = 2.45, SD = 0.63$) because students demonstrated strong ability to use formal symbolic representations during problem-solving. The indicator which required students to draw accurate conclusions from problem solving ($\bar{x} = 0.60, SD = 0.64$) received a low rating because students struggled to connect their solution steps to the proper final answers.

Ari wants to make a large dice out of paper for playing. He draws a net as shown in the following figure.

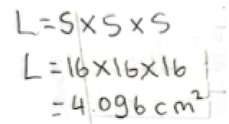
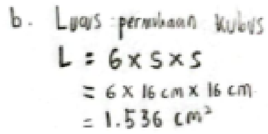
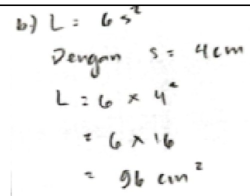


a. Calculate the surface area of the solid formed from the net.

Figure 7. Item No. 4b

For the indicator of identifying known information in the problem, the item is shown in Figure 7, and student responses are summarised in Table 8. Results indicate that 60.2% of students made conceptual errors (e.g., no response, calculating volume instead of surface area), 34.7% committed procedural errors (e.g., using the wrong formula, calculating the total height of the net as the side length), and only 5.1% were able to solve the problem correctly.

Table 8. Student Errors in Indicator 3a

Proficiency Category	Student Errors	Student Responses
Low	Conceptual errors (no response, calculating volume instead of surface area)	
Medium	Procedural errors (wrong formula, misinterpreting the total height of the net as side length)	 The surface area of cube L = $6 \times s \times s$ = $6 \times 16\ cm \times 16\ cm$ = $1.536\ cm^2$
High	None	 L = $6 \times s^2$ with s = $4\ cm$ L = 6×4^2 = 6×16 = $96\ cm^2$

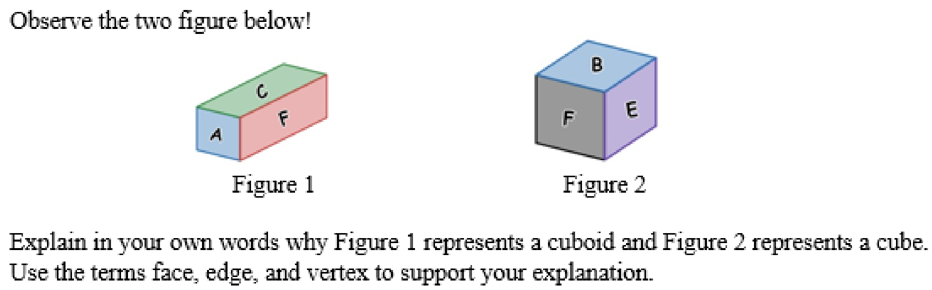
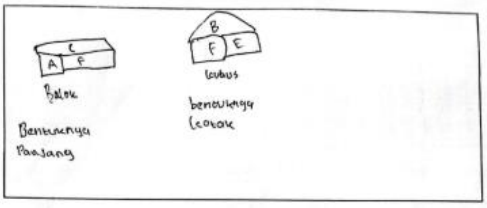
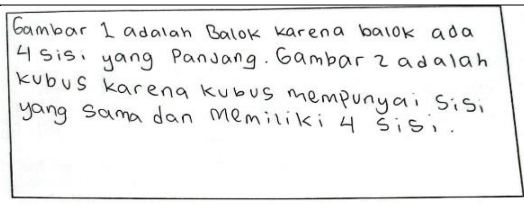
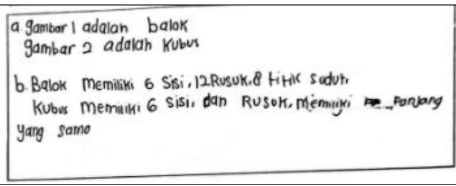


Figure 8. Item No. 3

For the indicator of expressing ideas about spatial objects or relationships accurately in written or oral form, the corresponding item is presented in Figure 8. The results, shown in Table 9, indicate that 63.6% of students made conceptual errors (e.g., no response, providing an answer without justification, giving incorrect reasoning, or failing to understand the formal definition of a cube/cuboid) and communication errors (e.g., not using terms such as face, edge, or vertex). Meanwhile, 28.8% demonstrated partial conceptual errors (e.g., mentioning terms such as face, edge, or vertex but without specifying quantity or type) and representational errors (e.g., knowing the general characteristics of a cube/cuboid but failing to explain the complete structure). Only 7.6% of students provided a correct and complete response.

Table 9. Student Errors in Indicator 3b

Proficiency Category	Student Errors	Student Responses
Low	Conceptual errors (no response, answer without reasoning, incorrect reasoning, failure to understand the formal definition of a cube/cuboid); communication errors (not using terms such as face, edge, vertex)	 Balok Bentuknya Panjang; kubus bentuknya kotak A cuboid Characterized by its elongated shape; A cube characterized by its box-like shape
Medium	Partial conceptual errors (mentioning face, edge, vertex without specifying number/type); representational errors (general characteristics identified but incomplete explanation)	 Gambar 1 adalah Balok karena balok ada 4 sisi. yang Panjang. Gambar 2 adalah kubus karena kubus mempunyai sisi yang sama dan memiliki 4 sisi. Figure 1 represents a cuboid because it has four longer faces. Figure 2 represents a cube because all of its faces are equal in size and it has four identical faces
High	None	 a. Gambar 1 adalah balok Gambar 2 adalah Kubus b. Balok memiliki 6 sisi, 12 Rusuk, 8 titik sudut. Kubus memiliki 6 sisi, dan Rusuk, memiliki panjang yang sama a. Figure 1 is a cuboid Figure 2 is a cube b. A cuboid has 6 faces, 12 edges, and 8 vertices c. A cube also has 6 faces, with all of its edges being equal in length

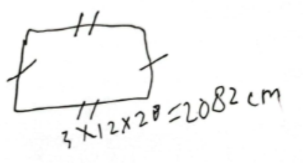
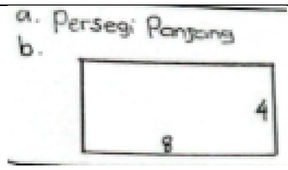
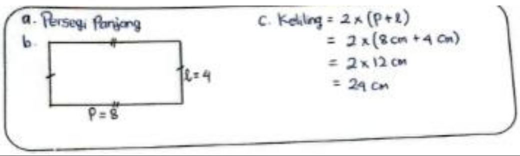
A two-dimensional figure has the following properties:
1. It has four sides
2. Two pairs of opposite sides are parallel and equal in length
3. All of its angles are right angles

The figure has a length of 8 cm and a width of 4 cm.
a. State the name of this figure using the correct mathematical term.
b. Draw a sketch of the figure, label the length with *p* and the width with *l*, and write the side lengths using the unit symbol cm.
c. Determine the perimeter of the figure using the correct mathematical formula, including proper notation and units.

Figure 9. Item No. 2

For the indicator of correctly using mathematical terms, notations, and symbols related to spatial objects or relationships, the results are shown in Table 10. A total of 7.6% students made conceptual errors (e.g., no response, confusing perimeter with area), visualisation errors (e.g., inaccurate drawings), and notational errors (e.g., omission of units or formulas). Furthermore, 39.8% committed partial conceptual errors (e.g., naming the figure as "square" instead of "rectangle") and notational errors (e.g., producing a correct sketch but without notation, writing only side lengths, or reporting results without units). On the other hand, 52.6% of students answered correctly.

Table 10. Student Errors in Indicator 3c

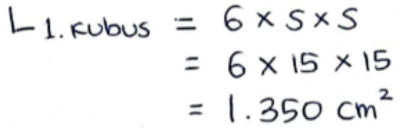
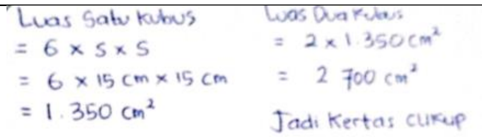
Proficiency Category	Student Errors	Student Responses
Low	Conceptual errors (no response, confusing perimeter and area); visualisation errors (inaccurate sketch); notational errors (omission of formula/units)	
Medium	Partial conceptual errors (misnaming figure, e.g., "square" instead of "rectangle"); notational errors (correct sketch but lacking notation or units)	 a. Rectangle b. [rectangle] 4, 8
High	None	 a. Rectangle b. [rectangle] $l = 4$, $p = 8$ c. perimeter = $2 \times (p + l)$ = $2 \times (8\ cm + 4\ cm)$ = $2 \times 12\ cm$ = $24\ cm$

Andi wants to wrap a gift box in the shape of a cube with a side length of 15 cm. He has a sheet of wrapping paper measuring $50\ cm \times 50\ cm$.
Is the wrapping paper sufficient to wrap two gift boxes of the same size?
Calculate the total area of paper required, and then write your conclusion in a clear sentence.

Figure 10. Item No. 8

For the indicator of drawing accurate conclusions from problem solving, the item is shown in Figure 10, and responses are summarised in Table 11. Results reveal that 91.5% of students committed communication errors (e.g., no response, simply writing "enough" without justification), spatial conceptual errors (e.g., calculating volume and comparing it to surface area), and conceptual errors (e.g., calculating the area of paper but directly concluding "enough" without computing the surface area of the cube). Another 8.5% made partial conceptual errors (e.g., correctly calculating the surface area of one cube but forgetting to multiply by two), communication errors (e.g., correctly calculating the surface area of two cubes but providing an unclear conclusion), or notational errors (e.g., correct calculation but omission of units). No students were able to provide a fully correct and complete answer.

Table 11. Student Errors in Indicator 3d

Proficiency Category	Student Errors	Student Responses
Low	Communication errors (no response, answer without justification); spatial conceptual errors (comparing volume with surface area); conceptual errors (calculating only paper area without cube surface area	 The surface area of one cube $= 6 \times s \times s$ $= 6 \times 15 \times 15$ $= 1.350\ cm^2$
Medium	Conceptual errors (correct surface area of one cube but not multiplied by two); communication errors (correct solution but conclusion not explicit); notational errors (no units included)	 The surface area of one cube $= 6 \times s \times s$ $= 6 \times 15 \times 15$ $= 1.350\ cm^2$ The surface area of two cubes $= 6 \times 1.350\ cm^2$ $= 2.700\ cm^2$ Therefore, the wrapping paper is sufficient

DISCUSSION

The research findings showed that elementary students exhibited different levels of spatial literacy in three-dimensional geometry because their visualisation abilities, reasoning skills, and communication competencies exhibited distinct patterns. The results showed that students face difficulties with conceptual and integrative aspects of spatial understanding despite their ability to perform basic procedural tasks. The development of spatial literacy at the elementary level remains incomplete because students demonstrate uneven progress across different areas.

Visualization Domain

The visualisation domain stands as the main obstacle which students need to overcome. Students performed poorly when they needed to create visual representations, transform nets into three-dimensional objects, or draw spatial objects. Students made various mistakes during the task, including failing to draw anything, drawing incorrect shapes, and creating sketches that lacked accuracy and failed to match each other. The students who succeeded in drawing cubes and cuboids were extremely rare. Students face major difficulties when they need to link two-dimensional representations to three-dimensional objects through spatial vision. Students who attempted to draw cubes failed to create cubes with equal-length edges and hidden faces. The students demonstrated understanding of 3D shape surface characteristics, but failed to recognise their geometric properties. Research from previous studies supports the complex nature of spatial vision, which develops through hands-on and digital learning experiences (Uttal, Miller, & Newcombe, 2013; Herrera, Ordóñez, & Ruiz-Loza, 2024). Research conducted in Indonesia shows that students achieve better geometric visualisation skills and demonstrate greater interest in learning when combining digital tools with culturally appropriate learning environments. Students develop better spatial understanding through interactive exploration of geometric objects in animated and augmented reality environments (Kurniawan, Nurfitriani, Setiyani, & Santi, 2025; Yanuarto & Iqbal, 2022). Research evidence demonstrates that students need various visual representations and interactive media to develop their visualisation skills, which leads to better conceptual understanding (Lowrie & Logan, 2023; Schoenherr, Strohmaier, & Schukajlow, 2024).

Reasoning Domain

The reasoning domain presented different patterns of performance in the assessment. The assessment results showed that students performed well in basic spatial concept comparison tasks but struggled with tasks that required multiple-step analysis and representation integration. Students demonstrated the ability to perform basic reasoning tasks, but their performance declined when they needed to understand multiple dimensions. Students who compared solids with different orientations used surface appearance instead of volume or congruence qualities to make their comparisons. The students based their reasoning on visual characteristics instead of structural elements. Research studies demonstrate that spatial reasoning consists of three separate components, which require students to use different mental abilities for learning (Fujita, Kondo, Kumakura, & Kunimune, 2020)(Schenck & Nathan, 2024). The geometric thinking framework developed by Van Hiele shows that most elementary students remain at visual or descriptive levels without advancing to analytical reasoning (Imami & Wafa, 2023) (Prayito, Suryadi, & Mulyana, 2019). Students need a structured educational system with step-by-step activities that help them advance from concrete understanding concrete concepts to understanding abstract ideas.

Communication Domain

The assessment results for communication skills showed significant variations between students. According to the assessment results, students demonstrated strong proficiency in using arithmetic symbols and terms. The students achieved a failure rate of more than 90% in drawing logical conclusions and explaining their reasoning, even though their mathematical calculations were correct. Students who correctly calculated the surface area of a cuboid failed to present their findings in written conclusions. The results show that students who excel at procedural tasks do not automatically develop strong conceptual communication abilities (Fauziyah & Jupri, 2020; Na'im & Mukhlis, 2024). According to Van Hiele's framework, students face challenges when they attempt to advance from descriptive reasoning to formal deductive communication. Research indicates that students develop better mathematical communication skills through verbal reflection, peer dialogue, and contextual discussion activities (Calkins, Grannan, & Siefken, 2020; Wong & Low, 2020). Classroom discussions that focus on "explaining why" rather than "showing how" will improve students' conceptual understanding of geometry through communication.

Integrated Insights and Implications

The research demonstrates that visualisation, reasoning, and communication form interconnected spatial literacy domains which function as a unified system. The research demonstrates that students who fail to visualise objects will also perform poorly in reasoning and communication tasks, which supports the concept of spatial literacy as a unified system (Ardelean, 2014) (Bancilhon, Wright, Ha, Crouser, & Ottley, 2023). The research evidence demonstrates that educational approaches should combine visual learning with activities that develop reasoning and communication competencies. The research evidence supports the development of a Local Instructional Theory (LIT), which teaches PMRI principles through exploration, modelling, and reflection activities using museum artefacts and digital geometry tools.

Practical Recommendations

Teacher should implement these methods to assist students in developing their visualization abilities and reasoning skills and communication competencies: (1) Teachers should assist students in developing visualization abilities through hands-on activities combined with digital tools including AR and animated geometry software, (2) Teachers should create structured problem sequences to help students organize spatial information for developing their multi-step reasoning abilities, (3) Teachers should assist students in developing conceptual communication abilities through peer discussion support and verbal explanation practice before introducing formal notation.

CONCLUSION

The research demonstrates how elementary school students develop spatial literacy through three-dimensional geometry by examining indicators of visualisation, reasoning, and communication. Students face difficulties when they try to link two-dimensional shapes to three-dimensional objects and when they engage in complex spatial reasoning and create accurate conceptual summaries. The study shows that students made numerous mistakes in visualisation and conceptual understanding, procedural operations, representational tasks, and communication activities, indicating that their learning difficulties stem from multiple complex factors. Students achieved higher grades when they completed fundamental procedural assignments that involved form comparison and the application and use of formal terms and notation. The assessment results demonstrate that students excel at basic procedural tasks, yet their conceptual understanding remains weak, posing a significant barrier to the development of spatial literacy at this grade level. The research establishes a Local Instructional Theory (LIT) for three-dimensional geometry education through its theoretical framework. The research identifies specific student mistakes in visualisation and reasoning and communication tasks, which enables developers to create targeted educational approaches for better results. The research establishes patterns of student spatial literacy while providing educational methods for teachers and theoretical frameworks for researchers to create effective spatial literacy instruction that connects procedural and conceptual abilities. Research should continue by developing new instructional methods based on the proposed Local Instructional Theory to assess their ability to enhance students' spatial literacy skills over multiple years. Research should investigate how students learn geometry through digital tools and cultural elements to create interactive learning experiences. Research studies that track student development of visualisation, reasoning, and communication skills from one grade to the next will help scientists understand how spatial literacy skills evolve in elementary school education.

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Author Contribution	:	Dina Octaria: Conceptualization, Writing - Original Draft, Formal Analysis, and Methodology; Zulkardi: Writing - Review & Editing, Methodology, Validation, Formal Analysis, and Supervision;

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