Edumatica: Jurnal Pendidikan Matematika
Volume 16 Number 1 April 2026

Submitted : 23 October 2025
Revised : 09 November 2025
Accepted : 13 December 2025

DOI: https://doi.org/10.22437/edumatica.vl6i1.49135

Designing a Project-Based Learning Worksheet: Validity and Feasibility Evidence from the Geometrocity Project

Fadhia Aliva Wibowo[1], Rohati Rohati[2*], Duano Sapta Nusantara[3], Ade Kumalasari[4]
[1,2,3,4]Universitas Jambi, Indonesia
E-mail*: rohati.fkip@unja.ac.id

Abstract

Students often struggle to express mathematical ideas through multiple external representations, particularly when learning polyhedral geometry that requires coordination of visual, verbal, and symbolic forms. This study aims to design a project-based learning worksheet and examine its validity and feasibility through the Geometrocity project. In this activity, students construct a miniature city as a contextual medium to explore geometric concepts while engaging with structured representational tasks. The study follows the ADDIE development framework and is limited to the development stage. The focus lies on producing a valid and feasible learning tool rather than measuring classroom effectiveness. Two material experts and one media expert evaluated the worksheet using validation instruments that assess content accuracy, language clarity, instructional presentation, and graphical design. Quantitative ratings were analyzed using descriptive statistics, while qualitative feedback supported iterative product revisions. The results indicate that the Geometrocity worksheet achieved high validity scores across all assessed aspects and met the criteria for feasibility. These findings suggest that the worksheet is suitable for use in further implementation studies. This study contributes a project-based worksheet that integrates structured representational activities within contextual geometry tasks.

Keywords: External Representation; Geometrocity; Mathematics; Project-Based Learning; Student Worksheet

INTRODUCTION

Mathematics education in junior high school plays an essential role in strengthening students' numeracy literacy and supporting their ability to interpret mathematical ideas across diverse contexts (Kolar & Hodnik, 2021). Effective mathematics learning extends beyond procedural competence and requires students to understand, communicate, and apply mathematical concepts in real situations (Siswantari et al., 2025). One key component that supports this process is representational ability, particularly external representation. External representation refers to the expression of mathematical ideas through diagrams, graphs, symbols, models, and written explanations (Ahmad et al., 2023; Duval, 2006). These representations enable learners to articulate reasoning processes and communicate mathematical thinking in a clear and structured manner (Abrahamson et al., 2020). Moreover, external representations function as cognitive tools that transform abstract concepts into forms that are more concrete and accessible for learners (Dreher et al., 2024).

From a theoretical perspective, Duval's theory of semiotic representation registers highlights that mathematical understanding emerges through the coordination of multiple forms of representation. According to this framework, conceptual learning requires students to translate and connect visual, symbolic, and verbal representations. Difficulties in shifting between these representational registers may limit students' conceptual development and restrict their ability to construct meaning from mathematical ideas (Uegatani et al., 2024). This perspective suggests that instructional design should deliberately provide opportunities for students to engage with and coordinate multiple representations during learning activities (Desai et al., 2021).

Despite its importance, many studies report persistent difficulties among students in constructing and using appropriate representations during problem solving. Kohen & Orenstein (2021) found that students frequently struggle to model real-world objects or produce accurate diagrams when solving mathematical tasks. Similar difficulties appear in symbolic reasoning, particularly when students must formulate mathematical expressions from contextual problems (Hidayat & Mawaddah, 2023). Hughes, Riccomini, & Lee (2020) also reported that students often encounter challenges when communicating mathematical reasoning using symbols or written explanations. Recent research further indicates that representational barriers remain common among lower secondary students, especially when they must transition between symbolic and visual representations in geometry and modeling tasks (Hatisaru, 2020).

These challenges align with broader findings showing that junior high school students tend to rely heavily on procedural strategies rather than constructing conceptual understanding through multiple representations. Such tendencies often result in weak performance when students encounter contextual or non-routine problems (Ahmar & Soro, 2023; Kablan & Uğur, 2020). In addition, several studies have highlighted limitations in existing instructional materials. Many student worksheets emphasize procedural exercises but rarely provide explicit opportunities for students to develop visual, symbolic, and verbal representations simultaneously. Astuti et al., (2022) observed that worksheets frequently fail to guide students in constructing representations systematically. Similar conclusions were reported by Gradini & Susanti (2022) as well as Sukmaningthias et al., (2022) who noted that the absence of representational scaffolding often leads to fragmented conceptual understanding.

Empirical evidence from a diagnostic test conducted in this study confirms these findings. Results from ninth-grade students show relatively higher performance on visual representation tasks (69.4%) compared with tasks that require verbal explanations (33.3%) and symbolic expressions (40.3%). These results indicate that students are more comfortable interpreting visual forms but encounter difficulties when they must articulate reasoning or express ideas symbolically. Similar patterns have been reported in broader educational contexts, where limited opportunities to externalize mathematical reasoning often led students to rely on superficial procedural approaches (Magana et al., 2024). These findings highlight the need for instructional innovations that deliberately support the development of multiple representations.

Contextual learning environments have shown potential to improve students' reasoning and engagement in mathematics. Marhamah et al., (2024) reported that contextual problem situations enhance students' reasoning processes, while Susanti et al., (2024) identified improvements in motivation and conceptual understanding when learning activities incorporate real-life contexts. However, research also indicates that contextualization alone does not guarantee deep conceptual learning. Cai & Gu (2022) argue that visual tools without explicit representational integration may fail to support meaningful understanding. Therefore, contextual tasks must be combined with structured representational activities to support conceptual development. Recent innovations also emphasize the role of local contexts in strengthening reasoning processes by connecting mathematical ideas with students' real-life experiences (Nusantara et al., 2025).

Project-Based Learning (PjBL) provides a promising pedagogical approach to address these challenges. This model engages students in meaningful learning through authentic tasks and collaborative problem solving (Chang et al., 2024; Kokotsaki et al., 2016). Recent studies show that PjBL can enhance student autonomy, engagement, and higher-order thinking skills (Himmi et al., 2025; Yang et al., 2022). Additional research also reports improvements in creative problem solving (Loyens, van Meerten, Schaap, & Wijnia, 2023), scientific creativity (Markula & Aksela, 2022), and student motivation (Maros, Korenkova, Fila, Levicky, & Schoberova, 2023). Within project-based environments, students often interact with multiple forms of representation when designing solutions, constructing models, and communicating ideas. Pramasdyahsari, Setyawati, Wardani, Fenyvesi, & Jusni (2025) demonstrated that project-based tasks can facilitate representational coordination, while Magana et al. (2024) and Song et al., (2024) reported improvements in representational reasoning through design-oriented projects. Similar findings were documented in geometry learning contexts (Žakelj & Klančar, 2022).

Research on PjBL-based instructional materials has also expanded in recent years. Project-based worksheets have been shown to enhance student engagement and learning autonomy (González-Ramírez & García-Hernández, 2022). Contextualized worksheets further support meaningful learning by connecting mathematical tasks with students' everyday experiences (López & Palacios, 2024). Sudianto et al., (2023) report improvements in mathematical literacy among pre-service teachers and enhanced critical thinking among junior high school students (Kemdikbudristek, 2023). Nevertheless, most existing studies primarily focus on general outcomes such as student participation, motivation, or collaboration. Few investigations explicitly examine how PjBL-based worksheets can support the development of students' external representation skills.

This limitation indicates a significant research gap. Instructional materials that intentionally integrate structured scaffolds for visual, symbolic, and verbal representations within project-based learning remain limited, particularly in the context of geometry learning at the junior high school level. Addressing this gap requires the development of learning materials that combine contextual project activities with explicit representational guidance.

Responding to this need, the present study introduces the Geometrocity Project, a project-based worksheet designed to foster students' external representation skills in geometry. In this project, students construct a miniature city by creating two-dimensional plans, developing three-dimensional models, performing symbolic calculations, and providing written explanations of geometric concepts. The worksheet integrates representational scaffolds based on semiotic representation theory, ensuring that representational processes are embedded systematically within the project activities.

Previous studies have demonstrated the potential of project-based learning to enhance higher-order thinking and mathematical communication Yanto et al., (2020) while recent design-oriented innovations such as pocketbook and augmented reality media have also shown potential to strengthen conceptual and creative thinking in mathematics (Nusantara et al., 2025). However, research that specifically integrates representational scaffolds within PjBL-based worksheets for geometry learning remains limited. Therefore, this study offers a novel contribution by designing and evaluating a project-based worksheet that explicitly targets the development of students' external mathematical representations.

Accordingly, this study aims to design and examine the validity and feasibility of a PjBL worksheet, the Geometrocity Project, to support junior high school students' external representation skills in geometry. The study contributes theoretically by demonstrating how project-based learning can be structured to strengthen representational processes in mathematics education. Practically, it provides a validated instructional tool that addresses students documented difficulties in coordinating multiple representations. In a broader perspective, this research also supports the goals of Sustainable Development Goal (SDG) 4: Quality Education, particularly in promoting inclusive and high-quality learning resources that strengthen numeracy literacy and critical thinking skills among secondary school students.

METHOD

This study employed a research and development (R&D) approach to design and validate a Project-Based Learning (PjBL) student worksheet. The development process followed the ADDIE framework, which includes five stages: analysis, design, development, implementation, and evaluation (Branch, 2009; Plomp & Nieveen, 2014).

The present study focused on the first three stages: analysis, design, and development. This scope aligns with the objective of producing a valid and feasible instructional product before conducting large-scale classroom implementation. Limiting development studies to the early stages of ADDIE is common in educational research when the main goal is to ensure product validity through expert review prior to field testing (Creswell & Creswell, 2018; Sari et al., 2024). The product developed in this study was a Project-Based Learning worksheet entitled The Geometrocity Project, designed to support junior high school students' external mathematical representation skills in learning polyhedral geometry.

The research was conducted at a public junior high school in Jambi City, Indonesia. Two groups of participants were involved. First, expert validation involved two validators: one mathematics education expert and one instructional media expert. Both were selected purposively based on the following criteria: holding at least a master's degree in mathematics education or educational technology, having a minimum of five years of professional experience, and possessing prior experience in evaluating instructional materials. Second, practicality testing involved 32 ninth-grade students and one mathematics teacher who used the worksheet during regular classroom learning.

The development procedure followed three stages of the ADDIE model: analysis, design, and development. This procedure provided a systematic framework for designing and refining the instructional material. The overall research workflow is presented in Figure 1.

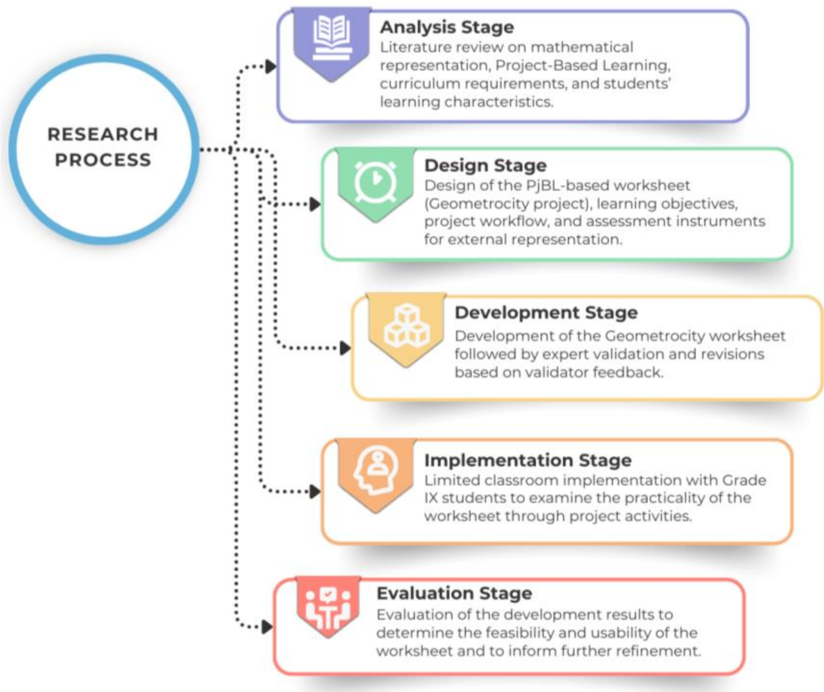

Figure 1. Research Procedure based on the ADDIE development model

As shown in Figure 1, the research process began with the analysis stage to identify instructional needs and examine students' difficulties in external mathematical representation. The findings from this stage served as the basis for determining learning objectives and designing the structure of the worksheet and project activities. The subsequent design stage focused on organizing the learning tasks according to the Project-Based Learning framework. Finally, the development stage involved producing the initial worksheet prototype, conducting expert validation, and revising the material before limited classroom implementation. The detailed procedures undertaken in each stage are described as follows.

Analysis Stage

This stage aimed to identify instructional needs through curriculum analysis, examination of student characteristics, and a diagnostic assessment of students' external representation skills. The diagnostic test consisted of three items measuring visual, verbal, and symbolic representations. The results indicated that students performed better on visual tasks but experienced difficulties expressing mathematical ideas verbally and symbolically. These findings informed the formulation of learning objectives and representational scaffolding within the worksheet.

Design Stage

The worksheet structure was designed according to the Project-Based Learning syntax, including driving questions, project planning, scheduling, monitoring, testing results, and reflection (Kemdikbudristek, 2023). Each project activity required students to express mathematical ideas through drawings, physical models, symbolic calculations, and written explanations. This design aimed to promote coordination between visual, symbolic, and verbal representations, which is essential for conceptual understanding in mathematics learning (Pramasdyahsari et al., 2025; Song et al., 2024).

Development Stage

During this stage, the initial worksheet prototype was produced and evaluated through expert validation and practicality testing. Experts assessed the quality of the worksheet prior to classroom use. After revision, the worksheet was implemented in a small-scale classroom setting to evaluate its usability.

Two instruments were used to collect data: validation questionnaires and practicality questionnaires. The validation questionnaire was completed by expert validators to assess four aspects of the worksheet: content accuracy, language clarity, instructional presentation, and graphical design (Nurhayati et al., 2022). Each indicator was rated using a five-point Likert scale ranging from 1 (very poor) to 5 (very good). The practicality questionnaire was administered to students and the mathematics teacher after using the worksheet. The instrument evaluated ease of use, clarity of instructions, usefulness for learning, and student engagement. Open-ended questions were also included in both instruments to obtain qualitative feedback for product revision.

Quantitative data obtained from the validation questionnaires were analyzed using descriptive statistics. The validity score was calculated by converting the obtained score into a percentage of the maximum possible score using the following formula:

$$\text{V} = \frac{\sum s}{N} \times 100\%$$

where V represents the validity index, $\sum s$ is the total score obtained, and N is the maximum possible score. The resulting percentage was then interpreted using the validation criteria presented in Table 1.

Table 1. Validation Criteria for the Geometrocity PjBL Worksheet

Validity Level	Validity Criteria
$85{,}01\% - 100{,}00\%$	Very valid, or can be used without revision
$70{,}01\% - 85{,}00\%$	Quite valid, or can be used with minor revisions
$50{,}01\% - 70{,}00\%$	Less valid, or not recommended for use because major revisions are required
$01{,}00\% - 50{,}00\%$	Invalid, or should not be used

Table 1 presents the validity criteria used to interpret the validation results. A score above 70% indicates that the material is at minimum quite valid and suitable for use with minor revisions.

Similarly, the practicality score was calculated by converting the total score obtained from students and the teacher into a percentage using the following formula:

$$P = \frac{\sum s}{N} \times 100\%$$

where P represents the practicality index, $\sum s$the total score obtained from the respondents, and N is the maximum possible score. The resulting percentage was interpreted according to the practicality criteria presented in Table 2.

Table 2. Practical Criteria for the Geometrocity PjBL Worksheet

Validity Level	**Validity Criteria**
$81\% \leq P \leq 100\%$	Very practical or usable without modification
$61\% \leq P \leq 81\%$	Quite practical, usable, needs minor improvements
$41\% \leq P \leq 61\%$	Not very practical, not recommended for use
$21\% \leq P \leq 41\%$	Not practical or not allowed to be used
$0\% \leq P \leq 21\%$	Very impractical, unusable

Table 2 presents the practical criteria used to evaluate teacher and student responses. A score of 81% or above indicates that the worksheet is very practical and can be used without modification.
The resulting percentages were interpreted using criteria adapted from (Akbar, 2013). Qualitative feedback from experts, teachers, and students was analyzed thematically and used to revise the worksheet prototype.

RESULTS

Analysis

The analysis phase examined instructional needs through curriculum analysis, student characteristics, and the initial condition of students' external representation skills. Curriculum analysis was conducted to ensure that the instructional material aligns with the learning outcomes specified in the merdeka curriculum for junior high school geometry. this process involved identifying the relevant learning outcomes and translating them into more operational learning objective flow that could guide the design of the project-based worksheet. The results of the curriculum mapping are summarized in Table 3.

Table 3. *Alignment of Learning Outcomes and Learning Objective Flow for Polyhedral Geometry*

Curriculum Component	**Relevance to the Study**
Curriculum Learning Outcomes	Students are expected to understand two- and three-dimensional geometric concepts and perform calculations involving perimeter, area, surface area, and volume. The curriculum also emphasizes the use of visual representations to solve contextual mathematical problems.
Learning Objective Flow	Learning activities guide students to construct, draw, and calculate geometric objects within real-world contexts, such as the Geometrocity project. Students are also encouraged to communicate their solutions through multiple forms of representation, including visual models, symbolic calculations, and written explanations.

As shown in Table 3, the curriculum emphasizes not only conceptual understanding of polyhedral geometry but also the ability to express mathematical ideas through different forms of representation. These competencies provide a foundation for designing learning activities that integrate visual modeling, symbolic reasoning, and verbal explanation within project-based tasks.

Following the curriculum analysis, a diagnostic test was administered to examine students' initial external representation skills. The test consisted of three items representing visual, verbal, and symbolic or mathematical expression indicators and was given to nine ninth-grade students. Student responses were evaluated using an analytic rubric that assessed accuracy, completeness, and clarity of representation. The results indicate that students achieved an average score of 69.4% for visual representation, while lower scores were obtained for verbal representation (33.3%) and symbolic representation (40.3%). These findings suggest that students are relatively capable of illustrating geometric objects visually but encounter substantial difficulty when expressing mathematical ideas through written explanations and symbolic formulations (Hatisaru, 2020).

Given the limited number of participants (n = 9), these findings are not intended to provide statistical generalization but rather to serve as a preliminary needs assessment for guiding product development. Nevertheless, the pattern of representation difficulties is consistent with previous international studies. According to Duval (2006), students frequently struggle to coordinate transformations between semiotic representation registers, particularly when translating visual information into symbolic form. Similarly, Dreher et al. (2024) report that representational reasoning develops slowly when instructional tasks do not explicitly require students to transform ideas across multiple representational forms (Lockwood & Ellis, 2022).

Based on these findings, the Project-Based Learning approach was selected as the instructional framework for the worksheet. Project-based environments provide opportunities for students to construct authentic products through collaborative investigation and reflection (Leow & Neo, 2023). Previous research has demonstrated that such environments can strengthen students' representational communication and conceptual understanding in mathematics learning (Markula & Aksela, 2022; Song et al., 2024). Therefore, the results of the analysis phase provide both empirical and theoretical justification for developing a PjBL-based worksheet that facilitates visual, verbal, and symbolic representations in spatial geometry learning.

Design

The design stage generated the initial form of the Project-Based Learning worksheet (Geometrocity), which was structured to align with the characteristics of junior high school learners and the geometry learning outcomes in the Merdeka Curriculum. Rather than merely providing tasks, the worksheet was intentionally designed to facilitate the development of students' external representation skills. This orientation shaped the selection of learning flow, activities, and visual, verbal, symbolic prompts embedded throughout the worksheet. The preliminary version of the Geometrocity worksheet developed during this stage can be accessed through the following link: https://bit.ly/GeometrocityWorksheet.

The resulting design consists of four main components: (1) an introductory section that guides students on how to navigate the project; (2) project activity sheets aligned with the PjBL syntax adopted from the Ministry of Education and Culture's model as cited in Kemdikbudristek (2023) including driving questions, project planning, scheduling, monitoring, product preparation, and reflection; (3) explicit representation tasks prompting students to draw geometric nets and 3D models, explain them verbally, and translate them into mathematical expressions; and (4) a reflection section where students evaluate their work and reasoning. Each project stage was structured to encourage active knowledge construction through model-building, discussion, validation of ideas, and written communication reflecting the principles of PjBL in supporting deeper conceptual understanding (Markula & Aksela, 2022; Song et al., 2024; Wu, 2024).

Unlike the procedural worksheets commonly used in mathematics classes, the *Geometrocity* design emphasizes guided exploration and representation. Space is provided for students to sketch models, write explanations, and represent dimensions symbolically, allowing smooth transitions across visual, verbal, and symbolic forms of external representation (Abrahamson et al., 2020). This approach is consistent with Duval (2006) theory that conceptual understanding in mathematics requires coordination between semiotic representation registers. The design also adheres to established instructional design principles such as clarity of visual layout, consistency of symbols, and minimization of unnecessary decorative elements to ensure readability and cognitive efficiency during problem-solving.

Overall, the design phase successfully produced a structured and representation-oriented worksheet ready for expert validation. The organization of activities and prompts is intended to support not only product completion in PjBL but also the process of expressing mathematical meaning through multiple representational forms, as recommended in recent studies on mathematics learning materials that foster external representation (Dreher et al., 2024; López & Palacios, 2024).

Development

The development phase aimed to finalize the Geometrocity Project-Based Learning (PjBL) worksheet through expert validation and iterative revision (Shakeel, Al Mamun, & Haolader, 2023). Two experts participated in the validation process: a mathematics education expert who evaluated the accuracy and relevance of the learning content, and a media design expert who assessed visual presentation, layout consistency, and readability. Both experts used a five-point Likert scale to evaluate the worksheet. The validation results are summarized in Table 4.

Table 4. *Material and Media Expert Validation Results*

No	Assessed Aspect	Validator 1	Validator 2	Average	Category
1	Content	80 %	81 %	80,5 %	Valid
2	Design (Media)	86,67 %	85 %	85,83 %	Very Valid
Overall average				**83,17 %**	**Very Valid**

As shown in Table 4, the worksheet obtained an overall validity score of 83.17%, indicating that the instructional content, language clarity, activity sequence, and visual presentation meet the feasibility standards for instructional materials. The design aspect received a slightly higher score than the content aspect, suggesting strong alignment between graphical presentation and pedagogical objectives. Comparable validity levels have been reported in previous studies on PjBL-based instructional materials, which highlight the role of clear visual design and contextual tasks in supporting effective mathematics learning(Nusantara, Pasaribu, et al., 2025).

Despite the high validity score, the validators provided constructive recommendations for refinement, particularly simplifying complex instructions, improving layout proportionality, and adding a reflection component. These suggestions were incorporated to ensure that the worksheet supports not only content mastery but also accessibility and ease of implementation for teachers and students. Following the expert validation results, several revisions were made to improve the clarity, instructional quality, and visual organization of the PjBL-based Geometrocity worksheet. The details of each revision are presented sequentially in Figures 2 through 9.

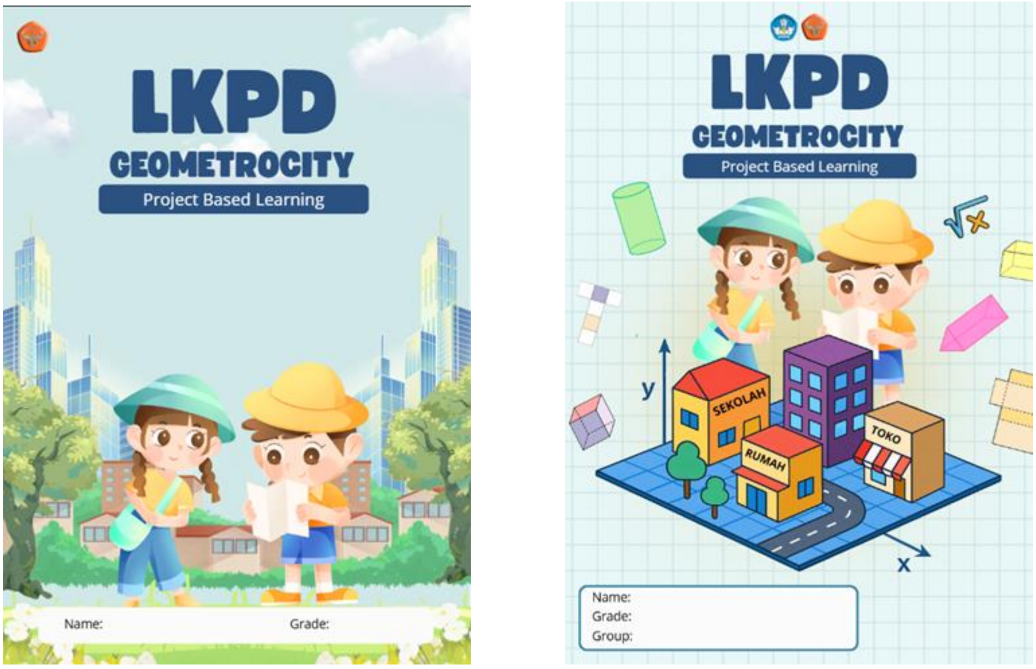

Figure 2. Improvement of Visual Layout and Proportionality (Before vs. After)

The first revision focused on increasing the visual proportionality and readability of the cover page. In the before version, the font size hierarchy, spatial margins, and image placement were inconsistent, which risked distracting students. In the after version, a more balanced layout was applied by standardizing title typography, centralizing the geometric project icon, and adjusting color contrast to improve accessibility for students. These changes were intended to support student engagement and reduce visual load while maintaining the identity of the Geometrocity project. The revised learning objective layout is illustrated in Figure 3.

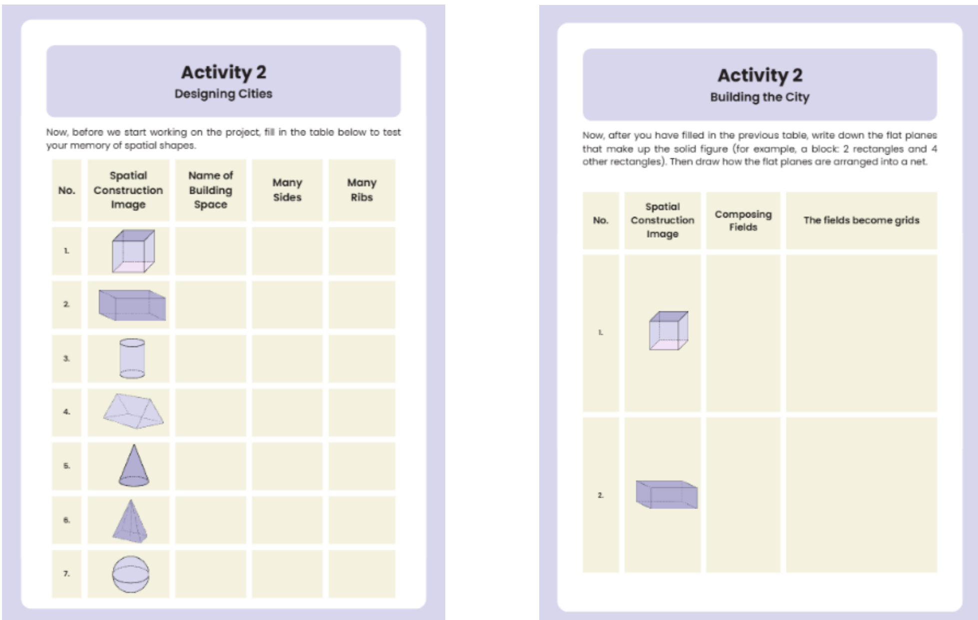

Figure 3. Refinement of Learning Objectives and Concept Focus (Before vs. After)

The second revision addressed the clarity and flow of the learning objectives related to nets of polyhedra. Initially, the objectives combined conceptual and procedural elements in a way that could overwhelm students. The revised version reorganizes the statements into a gradual progression from identifying polyhedron attributes to constructing nets enabling students to understand the expectations of the lesson more intuitively. This restructuring supports a more coherent learning pathway. The alignment improvement is shown in Figure 4.

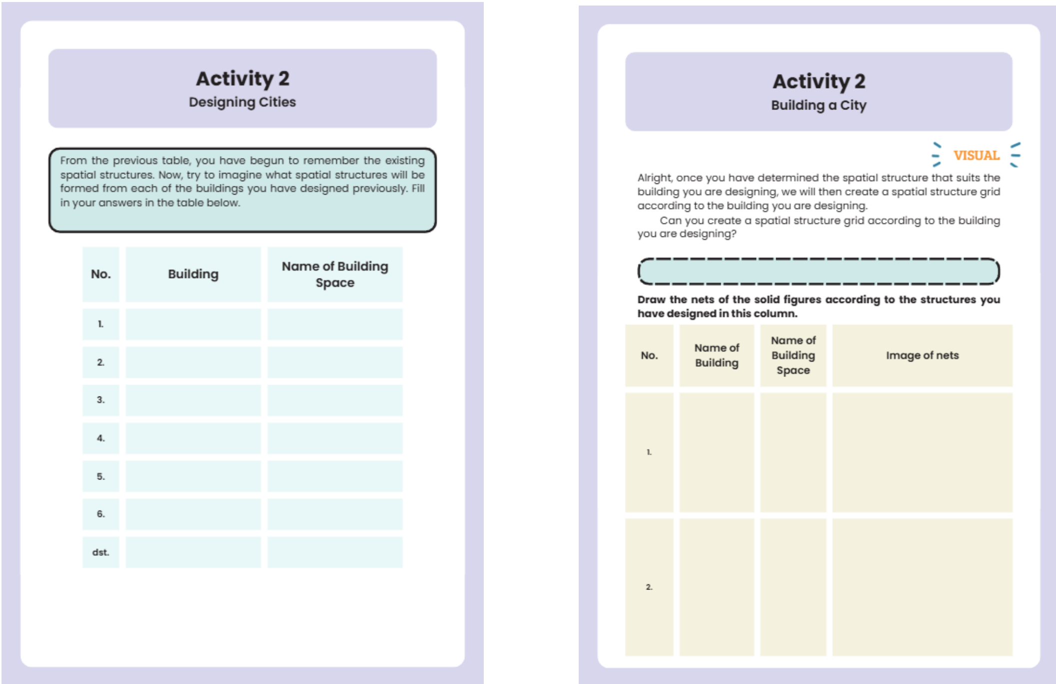

Figure 4. Strengthening Alignment Between Objectives and Activity Design (Before vs. After)

In the third revision, the alignment between learning objectives and activity tasks was made explicit. In the before version, the connection between activity steps and intended learning outcomes was not clearly visible to students. The after version links each activity directly to targeted indicators of external representation, helping students understand how every project task contributes to building visual, verbal, and symbolic mathematical reasoning. The revised sequencing is displayed in Figure 5.

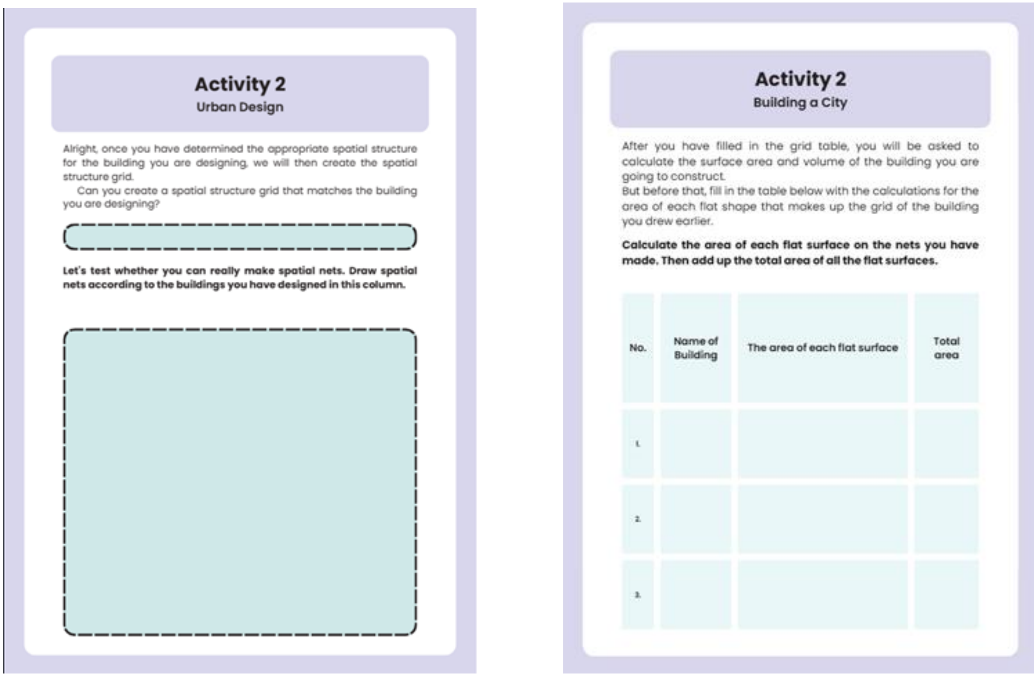

Figure 5. Integration of Polyhedron Analysis Before Net Construction (Before vs. After)

The fourth revision focused on mathematical sequencing. The earlier version positioned the construction of nets without sufficient preliminary analysis of polyhedron components. In the revised version, students first identify faces, edges, and vertices, and only then proceed to construct nets. This modification ensures that net construction is grounded in conceptual understanding rather than trial-and-error. The revised net construction sequence is shown in Figure 6.

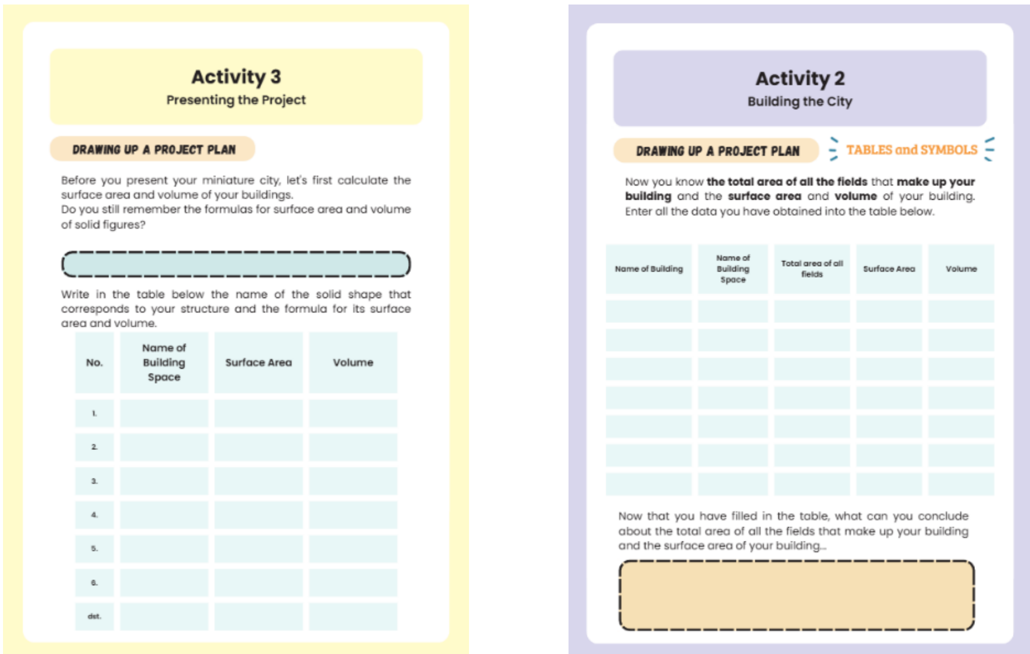

Figure 6. Reinforcement of Conceptual Link Between Nets and Surface Area (Before vs. After)

The fifth revision clarified the connection between building nets and calculating surface area. The before layout separated these two components, which risked students viewing them as unrelated tasks. The after version presents surface area calculations immediately after the net diagrams, prompting students to use their constructed visual representations to support symbolic computation. This improvement is presented in Figure 7.

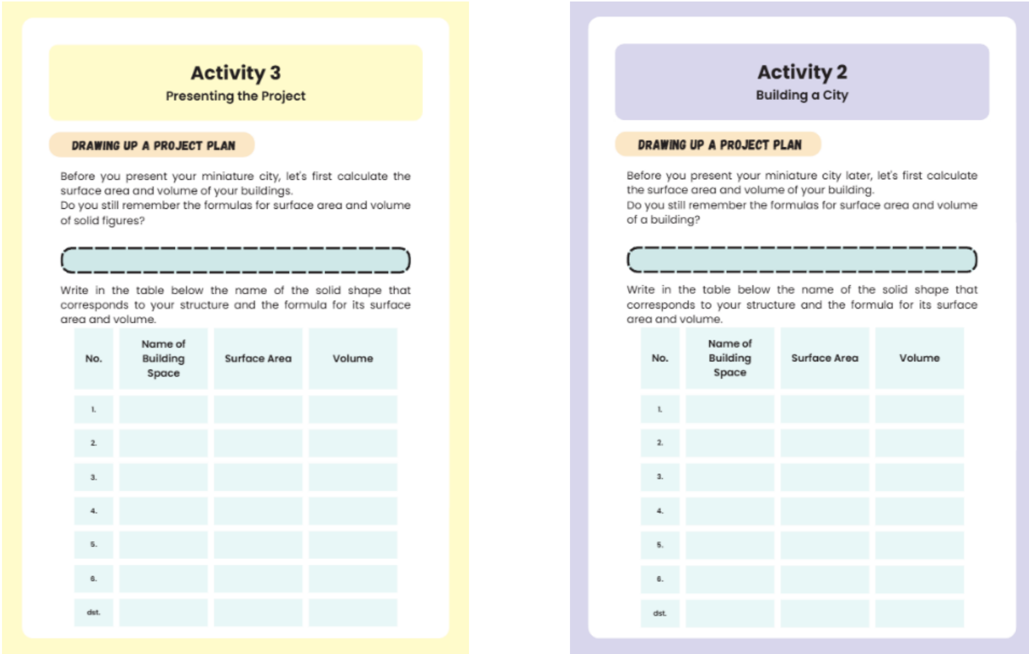

Figure 7. Reordering of Surface Area and Volume Tasks (Before vs. After)

The sixth revision improved learning progression by adjusting the sequence of geometric problem solving. Initially, surface area and volume appeared in a single task, which could burden students cognitively. The revised version separates them: surface area problems appear first, while volume problems are addressed in the next activity. This structure helps students internalize one concept securely before moving to the next. The reordered task layout is illustrated in Figure 8.

Figure 8. Enhancement of Contextual Problem Placement and Illustration (Before vs. After)

The seventh revision optimized the placement of the contextual 3D geometry task. Previously, the task appeared at a later stage without strong contextual cues. The revised version places the contextual prompt earlier and pairs it with a more relevant and motivating illustration. This framing encourages students to recognize the real-world relevance of geometry before engaging in symbolic work. The adjusted contextual task placement is shown in Figure 9.

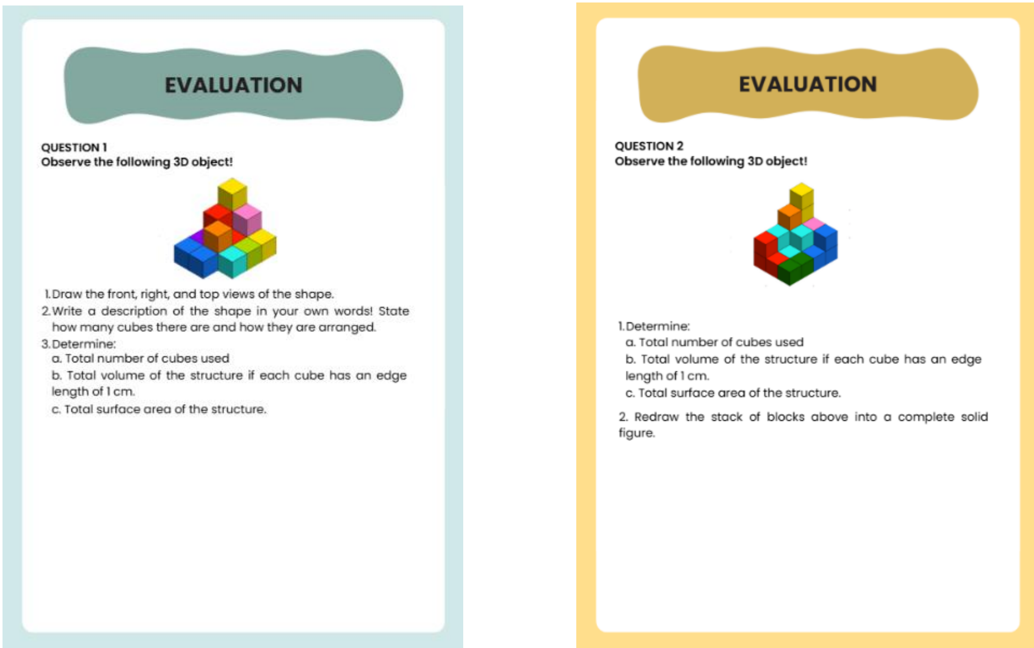

Figure 9. Adjustment of Drawing and Representation Tasks (Before vs. After)

The final revision improved the pacing of representational tasks. In the before version, students were required to draw geometric models too early, which risked producing sketches based on intuition rather than mathematical reasoning. In the after version, drawing is placed at the final phase of the activity after students have performed analysis, calculations, and planning. This encourages more accurate and meaningful external representations grounded in prior reasoning.

After revision based on expert input, the practicality test was conducted involving one mathematics teacher and 32 ninth-grade students to evaluate usability during learning activities. Practicality was assessed using a 5-point Likert scale and analyzed using Akbar (2013) practicality criteria. The results are presented in Table 5.

Table 5. *Practicality Test Results*

No	Respondent	Practicality Percentage	Category
1	Teacher	97,14 %	Very Practical
2	Student	86,29 %	Very Practical
Overall Average		91,72 %	Very Practical

As shown in Table 5, both teacher and student responses indicate a high level of practicality. The teacher reported that the structured sequence of activities supported the implementation of project-based learning and facilitated classroom management. Students also indicated that the worksheet was visually engaging, easy to follow, and helpful in completing the Geometrocity project collaboratively.

Overall, the development stage resulted in a worksheet that met both validity and practicality standards. The high validation score confirmed that the content and graphical design were feasible, while the practicality test indicated strong acceptance and usability by both teachers and students (Zardari, Hussain, Arain, Rizvi, & Vighio, 2021). With expert-driven revisions and positive user responses, the final version of the Geometrocity worksheet is ready for the next phase of implementation to further investigate its contribution to learning processes and students' external representation skills.

DISCUSSION

The high validity score obtained from expert evaluation indicates that the ADDIE-based development process successfully produced a feasible instructional tool for learning polyhedral geometry. Beyond confirming feasibility, this result demonstrates how alignment between curriculum objectives, instructional design, and project-based learning activities can support meaningful mathematical learning (Zhao, Zhao, & Li, 2023). The strong content validity reflects the coherence between the worksheet's learning trajectory and the competencies outlined in the Merdeka Curriculum. This finding is consistent with studies by Akbar (2013) and Najah (2024), which emphasize that effective instructional materials should demonstrate conceptual accuracy and be appropriate for students' cognitive development.

In the Geometrocity worksheet, project tasks were intentionally structured to guide students in interpreting geometric concepts through multiple forms of external representation. This approach aligns with Duval (2006) theory of semiotic representation, which states that mathematical understanding emerges when learners coordinate transformations across visual, symbolic, and verbal representations rather than relying on a single representational mode. Similar conclusions have been reported by Nusantara & Putri (2025) who show that tasks requiring students to articulate and justify mathematical reasoning can strengthen representational coordination and argumentation skills.

In addition to the accuracy of the content, the quality of media design contributed substantially to the high validity classification. Experts evaluated the visual organization, readability, and layout structure as highly supportive of student engagement. These findings highlight the important role of visual communication in helping students interpret geometric concepts. Prior research has shown that visualization linked to meaningful contexts can assist learners in interpreting abstract mathematical structures and developing representational fluency (López & Palacios, 2024; Sinclair & Bruce, 2015). Moreover, the integration of visual, symbolic, and verbal representations in instructional materials can reduce cognitive load and facilitate conceptual reasoning when design elements are coherent and purposeful (Latifa et al., 2025; Ruamba et al., 2025).

Empirical studies also demonstrate that well-designed visual and linguistic elements in mathematics learning media can enhance spatial reasoning and improve student engagement during geometry instruction (Winarni, Hanim, Kumalasari, Marlina, & Rohati, 2023; Winarni, Kumalasari, Marlina, & Rohati, 2021). Therefore, the effectiveness of the Geometrocity worksheet cannot be attributed solely to its visual appeal, but rather to the deliberate integration of design features that support mathematical meaning-making. These results are consistent with findings by Yanto et al. (2020) and Marhamah et al. (2024), who highlight that contextual geometry tasks supported by visual representations can strengthen students' understanding of spatial relationships and representational structures.

Feedback from the expert validators also emphasized the importance of balancing exploratory project activities with clear instructional guidance. Several revisions were made to simplify instructional language, clarify the sequence of tasks, and position representational spaces more strategically within the worksheet. These adjustments were necessary to improve usability and ensure that students could navigate the project activities independently. This observation supports the conclusions of Dewi & Darussyamsu (2024) and Weinhandl et al. (2024), who reported that worksheets with clear language and coherent visual organization can enhance student autonomy and facilitate smoother classroom implementation. Although classroom implementation was not included in the present study, the pattern of revisions indicates that contextual project activities combined with multimodal representation opportunities have strong potential to support representational competence.

This potential is consistent with previous research demonstrating that Project-Based Learning environments promote conceptual understanding through collaborative exploration, visualization, and mathematical communication (Markula & Aksela, 2022; Wu, 2024). Large-scale studies also suggest that instructional tools requiring students to transition between symbolic, verbal, and visual representations encourage deeper conceptual processing compared with materials focused primarily on procedural practice (Kamid et al., 2022; Winarni et al., 2024). Similar findings have been reported in mathematics education studies showing that representational scaffolding within worksheets can improve students' reasoning and communication of mathematical ideas (Astuti et al., 2022; Sukmaningthias et al., 2022).

Despite these promising results, several limitations should be acknowledged. First, the validation process involved only two experts, which may limit the diversity of professional perspectives captured in the evaluation. Second, the diagnostic test used to inform the worksheet design was conducted with a relatively small sample of nine students. Consequently, the findings from the diagnostic assessment should be interpreted as preliminary indicators rather than broadly generalizable results. Third, because the study focused only on the development stage of the ADDIE model, the practical impact of the worksheet on students' external representation abilities has not yet been empirically examined. Future studies should therefore conduct classroom implementation and effectiveness testing to evaluate the extent to which the Geometrocity worksheet can improve students' representational competence and conceptual understanding in geometry.

Nevertheless, the study provides several important implications for mathematics education. The Geometrocity worksheet offers a structured project-based learning resource that can support polyhedral geometry instruction by encouraging students to engage with multiple forms of representation. Effective teacher facilitation remains essential, particularly in guiding students to connect visual, symbolic, and verbal representations during project activities (Žakelj & Klančar, 2022). In addition, the development workflow used in this study, combining the ADDIE framework with expert validation, can serve as a methodological reference for future instructional material development aimed at strengthening representational competence in mathematics learning.

This implication aligns with recent studies indicating that project-based instructional tools designed within contextual learning environments can enhance student engagement and conceptual understanding (Ahmar & Soro, 2023; Kablan & Uğur, 2020). Furthermore, bibliometric analyses show that research on representation-based innovation has become a major trend in mathematics education between 2020 and 2025, particularly in Indonesia (Qodaria et al., 2025). These findings suggest that continued development of representation-oriented learning materials remains a relevant direction for advancing mathematics education research and practice.

CONCLUSION

This study developed the Geometrocity Project-Based Learning (PjBL) worksheet as a representation-oriented instructional material for learning polyhedral geometry. The development process produced a worksheet that integrates visual, verbal, and symbolic representation tasks within project-based activities, allowing students to construct and communicate mathematical ideas through multiple representational forms. Expert validation results indicate that the worksheet meets pedagogical and design feasibility standards, suggesting that it is suitable for use as a learning resource to support students' representational reasoning in geometry. However, this study was limited to the development stage of the ADDIE model, and therefore the effectiveness of the worksheet in improving students' external representation skills has not yet been empirically tested in classroom settings. Future research should examine its impact through classroom implementation and experimental or quasi-experimental designs, while also involving a broader panel of experts and extending the design framework to other mathematical topics to further strengthen its contribution to representation-oriented mathematics learning.

ACKNOWLEDGMENTS

The author expresses sincere gratitude to the expert validators for their valuable insights and constructive suggestions throughout the development of the Project-Based Learning–based Geometrocity student worksheet. The author also acknowledges the support provided by the Penelitian Tesis Magister (PTM) program for funding this research.

DECLARATIONS

Author Contribution	:	Fadhia Aliva Wibowo: Conceptualization, Data Curation, and Writing - Original Draft; Rohati: Validation, Supervision, Visualization, and Project administration; Duano Sapta Nusantara: Validation, Supervision, Formal analysis, and Writing - Review & Editing; Ade Kumalasari: Methodology, Validation, and Writing - Review & Editing
Funding Statement	:	This research was funded and supported by *Basis Informasi Penelitian dan Pengabdian kepada Masyarakat (BIMA)* through the research grant agreement Number 27/UN21.11/PT.01.05/SPK/2025.
Conflict of Interest	:	The authors declare no conflict of interest.
AI Use Statement	:	We hereby confirm that no artificial intelligence (AI) tools or methodologies were utilized at any stage of this study, including during data collection, analysis, visualization, or manuscript preparation. All work presented in this study was conducted manually by the authors without the assistance of AI-based tools or systems
Additional Information	:	Additional information is available for this paper.

REFERENCES

Abrahamson, D., Nathan, M. J., Williams-Pierce, C., Walkington, C., Ottmar, E. R., Soto, H., & Alibali, M. W. (2020). The future of embodied design for mathematics teaching and learning. *Frontiers in Education*, *5*. doi:10.3389/feduc.2020.00147

Ahmad, Akhsani, L., & Mohamed, Z. (2023). The profile of students' mathematical representation competence, self-confidence, and habits of mind through problem-based learning models. *Infinity Journal*, *12*(2), 323–338. doi:10.22460/infinity.v12i2.p323-338

Ahmar, S., & Soro, S. (2023). Pengaruh penggunaan E-LKPD dalam pembelajaran matematika terhadap kemampuan penalaran peserta didik kelas VIII SMP. *Edumatica: Jurnal Pendidikan Matematika*, *13*(2). doi:10.22437/edumatica.v13i02.26352

Akbar, S. (2013). *Instrumen Perangkat Pembelajaran*. Bandung: Remaja Rosdakarya.

Astuti, N., Jana, P., & Marsyiam. (2022). Pengembangan LKS berbasis etnomatematika untuk memfasilitasi kemampuan representasi matematis. *Edumatica: Jurnal Pendidikan Matematika*, *12*(2). doi:10.22437/edumatica.v12i02.15814

Branch, R. M. (2009). *Instructional Design: The ADDIE Approach*. Springer Science & Business Media.

Cai, H., & Gu, X. (2022). Factors that influence the different levels of individuals' understanding after collaborative problem solving: the effects of shared representational guidance and prior knowledge. *Interactive Learning Environments*, *30*(4), 695–706. doi:10.1080/10494820.2019.1679841

Chang, Y., Choi, J., & Şen-Akbulut, M. (2024). Undergraduate students' engagement in project-based learning with an authentic context. *Education Sciences*, *14*(2). doi:10.3390/educsci14020168

Creswell, J. W., & Creswell, J. D. (2018). *Research Design: Qualitative, Quantitative, and Mixed Methods Approaches* (5th Edition). SAGE Publications, Inc.

Desai, S., Bush, S. B., & Safi, F. (2021). Mathematical representations in the teaching and learning of geometry: A review of the literature from the United States. *Electronic Journal for Research in Science & Mathematics Education*, *25*(4), 6–22.

Dewi, M. P., & Darussyamsu, R. (2024). Meta analisis validitas dan praktikalitas pengembangan LKPD sebagai media pembelajaran. *Jurnal Pendidikan Tambusai*, *8*(2). doi:10.31004/jptam.v8i2.14929

Dreher, A., Wang, T. Y., Feltes, P., Hsieh, F. J., & Lindmeier, A. (2024). High-quality use of representations in the mathematics classroom – a matter of the cultural perspective? *ZDM - Mathematics Education*, *56*(5), 965–980. doi:10.1007/s11858-024-01597-5

Duval, R. (2006). A cognitive analysis of problems of comprehension in a learning of mathematics. *Educational Studies in Mathematics*, *61*(1–2), 103–131. doi:10.1007/s10649-006-0400-z

González-Ramírez, T., & García-Hernández, A. (2022). Design and validation of a questionnaire to assess student satisfaction with mathematics study materials. *International Journal of Instruction*, *15*(1), 1–20. doi:10.29333/iji.2022.1511a

Gradini, E., & Susanti. (2022). Pengembangan perangkat pembelajaran berbasis model learning cycle 5E untuk memfasilitasi perubahan konseptual siswa dalam pembelajaran matematika. *Edumatica: Jurnal Pendidikan Matematika*, *12*(2). doi:10.22437/edumatica.v12i02.16933

Hatisaru, V. (2020). Exploring evidence of mathematical tasks and representations in the drawings of middle school students. *International Electronic Journal of Mathematics Education*, *15*(3), 1–21. doi:10.29333/iejme/8482

Hidayat, L. D., & Mawaddah, S. (2023). Effectiveness of realistic mathematics approach to increasing mathematical representation ability at SMP N 9 Purwokerto. *International Journal of Research in Mathematics Education*, *1*(2), 121–130. doi:10.24090/ijrme.v1i2.9139

Himmi, N., Armanto, D., & Amry, Z. (2025). Implementation of project-based learning (PjBL) in mathematics education: A systematic analysis of international practices and theoretical foundations. *Science Insights Education Frontiers*, *26*(2), 4305–4321. doi:10.15354/sief.25.or699

Hughes, E. M., Riccomini, P. J., & Lee, J. Y. (2020). Investigating written expressions of mathematical reasoning for students with learning disabilities. *Journal of Mathematical Behavior*, *58*. doi:10.1016/j.jmathb.2020.100775

Kablan, Z., & Uğur, S. S. (2020). The relationship between routine and non-routine problem solving and learning styles. *Educational Studies*, *47*(3), 328–343. doi:10.1080/03055698.2019.1701993

Kamid, K., Rohati, R., Hobri, H., Triani, E., Rohana, S., & Pratama, W. A. (2022). Process skill and student’s interest for mathematics learning: Playing a traditional games. *International Journal of Instruction*, *15*(3), 967–988. doi:10.29333/iji.2022.15352a

Kemdikbudristek. (2023). *Panduan Implementasi Pembelajaran Berpusat Pada Mahasiswa*. Jakarta: Dirjen Dikti.

Kohen, Z., & Orenstein, D. (2021). Mathematical modeling of tech-related real-world problems for secondary school-level mathematics. *Educational Studies in Mathematics*, *107*(1), 71–91. doi:10.1007/s10649-020-10020-1

Kokotsaki, D., Menzies, V., & Wiggins, A. (2016). Project-based learning: A review of the literature. *Improving Schools*, *19*(3), 267–277. doi:10.1177/1365480216659733

Kolar, V. M., & Hodnik, T. (2021). Mathematical literacy from the perspective of solving contextual problems. *European Journal of Educational Research*, *10*(1), 467–483. doi:10.12973/EU-JER.10.1.467

Latifa, U., Indrasari, P., Pramuditya, S. A., & Asnawati, S. (2025). Connecting concepts and representations in mathematics: A sistematic literature review. *International Journal of Contemporary Studies in Education (IJ-CSE)*, *4*(2), 100–113. doi:10.56855/ijcse.v4i2.1483

Leow, F. T., & Neo, M. (2023). Critical factors for enhancing students' collaborative learning experiences in a project-based connectivism learning environment. *International Journal of Learning, Teaching and Educational Research*, *22*(7), 388–410. doi:10.26803/ijlter.22.7.21

Lockwood, E., & Ellis, A. B. (2022). Two students' mathematical thinking and activity across representational registers in a combinatorial setting. *ZDM - Mathematics Education*, *54*(4), 829–845. doi:10.1007/s11858-022-01352-8

López, J. A., & Palacios, F. J. P. (2024). Effects of a project-based learning methodology on environmental awareness of secondary school students. *International Journal of Instruction*, *17*(1), 1–22. doi:10.29333/iji.2024.1711a

Loyens, S. M. M., van Meerten, J. E., Schaap, L., & Wijnia, L. (2023). Situating higher-order, critical, and critical-analytic thinking in problem- and project-based learning environments: A systematic review. *Educational Psychology Review*, *35*(39). doi:10.1007/s10648-023-09757-x

Magana, A. J., Arigye, J., Udosen, A., Lyon, J. A., Joshi, P., & Pienaar, E. (2024). Scaffolded team-based computational modeling and simulation projects for promoting representational competence and regulatory skills. *International Journal of STEM Education*, *11*(1). doi:10.1186/s40594-024-00494-3

Marhamah, M., Zulkardi, Putri, R. I. I., Susanti, E., & Nusantara, D. S. (2024). Design experiment pada pembelajaran bangun datar dengan pendekatan PMRI bagi siswa autis. *Indiktika: Jurnal Inovasi Pendidikan Matematika*, *6*(2), 348–360. doi:10.31851/indiktika.v6i2.15085

Markula, A., & Aksela, M. (2022). The key characteristics of project-based learning: how teachers implement projects in K-12 science education. *Disciplinary and Interdisciplinary Science Education Research*, *4*(1). doi:10.1186/s43031-021-00042-x

Maros, M., Korenkova, M., Fila, M., Levicky, M., & Schoberova, M. (2023). Project-based learning and its effectiveness: evidence from Slovakia. *Interactive Learning Environments*, *31*(7), 4147–4155. doi:10.1080/10494820.2021.1954036

Najah, K. (2024). The influence of mathematical beliefs on the mathematical representation ability of students in Class VII. *International Journal of Research in Mathematics Education*, *2*(1), 27–34. doi:10.24090/ijrme.v2i1.9328

Nurhayati, Suwartono, T., Saripudin, A., & Juwita, N. (2022). Applying web-based technology in developing student worksheets on writing local culture content drama script. *Theory and Practice in Language Studies*, *12*(2), 263–273. doi:10.17507/tpls.1202.07

Nusantara, D. S., Ibarra, F. P., Pasaribu, F. T., & Lusinda, U. (2025). Designing JUMPISA: A PISA-based jumping task to support students' statistical literacy using jambi context. *Journal of Instructional Mathematics*, *6*(2). doi:10.37640/jim.v6i2.2518

Nusantara, D. S., Pasaribu, F. T., Gustiningsi, T., & Abdullah, A. H. (2025). Developing mind mapping-based pocketbook with augmented reality to support creative thinking skills in pre-service mathematics teachers. *SJME (Supremum Journal of Mathematics Education)*, *9*(2), 297–317. doi:10.35706/sjme.v9i2.204

Nusantara, D. S., & Putri, R. I. I. (2025). Investigating students' mathematical argumentation ability in solving pisa-like mathematics tasks on change and relationships. *Mathematics Education Journal*, *19*. doi:10.22342/mej.v19i4.pp807-826

Plomp, Tj., & Nieveen, Nienke. (2014). *Educational design research*. *Netherlands Institute for Curriculum Development (SLO)*. Enschede: Netherlands Institue for Curriculum Development (SLO).

Pramasdyahsari, A. S., Setyawati, R. D., Wardani, T. I., Fenyvesi, K., & Jusni, E. (2025). Exploring pre-service mathematics teachers' numeracy development through art-integrated STEAM geometry: Balancing creativity and mathematical precision. *Journal on Mathematics Education*, *16*(4), 1365–1388. doi:10.22342/jme.v16i4.pp1365-1388

Qodaria, R., Rohati, R., & Marlina, M. (2025). Bibliometric mapping of research on mathematical representation ability in Indonesia (2020–2025). *Journal of General Education and Humanities*, *4*(4), 2287–2296. doi:10.58421/gehu.v4i4.770

Ruamba, M. Y., Sukestiyarno, Y. L., Rochmad, R., & Asih, T. S. N. (2025). The impact of visual and multimodal representations in mathematics on cognitive load and problem-solving skills. *International Journal of Advanced and Applied Sciences*, *12*(4), 164–172. doi:10.21833/ijaas.2025.04.018

Sari, E. M., Misdalina, M., & Septiati, E. (2024). Pengembangan LKPD berbasis project-based learning (pjbl)materi bangun ruang sisi datar untuk siswa kelas VIII. *Jurnal Cendekia: Jurnal Pendidikan Matematika*, *8*(1), 634–645. doi:10.31004/cendekia.v8i1.2196

Shakeel, S. I., Al Mamun, M. A., & Haolader, M. F. A. (2023). Instructional design with ADDIE and rapid prototyping for blended learning: validation and its acceptance in the context of TVET Bangladesh. *Education and Information Technologies*, *28*(6), 7601–7630. doi:10.1007/s10639-022-11471-0

Sinclair, N., & Bruce, C. D. (2015). New opportunities in geometry education at the primary school. *ZDM Mathematics Education*, *47*(3), 319–329. doi:10.1007/s11858-015-0693-4

Siswantari, Sabon, S. S., Listiawati, N., Wirda, Y., Zulkardi, & Riyanto, B. (2025). Bridging mathematics and communication: Implementing realistic mathematics education principles for skill development. *Journal on Mathematics Education*, *16*(2), 729–752. doi:10.22342/jme.v16i2.pp729-752

Song, X., Razali, A. B., Sulaiman, T., & Jeyaraj, J. J. (2024). Impact of project-based learning on critical thinking skills and language skills in EFL context : A review of literature. *World Journal of English Language*, *14*(5), 402–412. doi:10.5430/wjel.v14n5p402

Sudianto, Laelasari, & Julia, R. (2023). Enhancing pre-service mathematics teachers' mathematical literacy through project-based learning. *International Journal of Advance Research in Mathematics Education*, *1*(1), 36–45. doi:10.56916/ijr.v1i1.319

Sukmaningthias, N., Sari, N., Susanti, E., & Safitri, D. (2022). LKS berbasis RME menggunakan cabri 3d untuk koneksi matematis siswa pada bangun ruang bidang lengkung. *Edumatica: Jurnal Pendidikan Matematika*, *12*(03). doi:10.22437/edumatica.v12i03.21364

Susanti, N., Putri, R. I. I., Susanti, E., & Nusantara, D. S. (2024). Learning trajectory flat building material using batik besemah context for elementary school students. *Numerical: Jurnal Matematika Dan Pendidikan Matematika*, *8*(1), 105–114. doi:10.25217/numerical.v8i1

Uegatani, Y., Otani, H., Shirakawa, S., & Ito, R. (2024). Real and illusionary difficulties in conceptual learning in mathematics: comparison between constructivist and inferentialist perspectives. *Mathematics Education Research Journal*, *36*(4), 895–915. doi:10.1007/s13394-023-00478-6

Weinhandl, R., Mayerhofer, M., Houghton, T., Lavicza, Z., Kleinferchner, L. M., Anđić, B., … Hohenwarter, M. (2024). Enhancing user-centred educational design: Developing personas of mathematics school students. *Heliyon*, *10*(2). doi:10.1016/j.heliyon.2024.e24173

Winarni, S., Hanim, M., Kumalasari, A., Marlina, M., & Rohati, R. (2023). Pengembangan buku saku berbasis augmented reality pada materi bangun ruang untuk meningkatkan kemampuan spasial siswa. *AKSIOMA: Jurnal Program Studi Pendidikan Matematika*, *12*(4), 3561. doi:10.24127/ajpm.v12i4.8193

Winarni, S., Kumalasari, A., Marlina, M., & Rohati, R. (2021). Efektivitas video pembelajaran matematika untuk mendukung kemampuan literasi numerasi dan digital siswa. *AKSIOMA: Jurnal Program Studi Pendidikan Matematika*, *10*(2), 574. doi:10.24127/ajpm.v10i2.3345

Winarni, S., Simanjuntak, R. P., Marlina, M., Rohati, R., & Kumalasari, A. (2024). Pengembangan e-modul interaktif untuk mendukung algebraic thinking pada materi turunan fungsi aljabar. *AKSIOMA: Jurnal Program Studi Pendidikan Matematika*, *13*(3), 791. doi:10.24127/ajpm.v13i3.8540

Wu, X. Y. (2024). Unveiling the dynamics of self-regulated learning in project-based learning environments. *Heliyon*, *10*(5). doi:10.1016/j.heliyon.2024.e27335

Yang, D., Chen, P., Wang, H., Wang, K., & Huang, R. (2022). Teachers' autonomy support and student engagement: A systematic literature review of longitudinal studies. *Frontiers in Psychology*, *13*. doi:10.3389/fpsyg.2022.925955

Yanto, Y., Zulkardi, Putri, R. I. I., & Nusantara, D. S. (2020). Learning geometry transformations using batik motifs: systematic literature review. *Histogram: Jurnal Pendidikan Matematika*, *8*(1), 160–168. doi:10.31100/histogram.v8i1.3337

Žakelj, A., & Klančar, A. (2022). The role of visual representations in geometry learning. *European Journal of Educational Research*, *11*(3), 1393–1411. doi:10.12973/eu-jer.11.3.1393

Zardari, B. A., Hussain, Z., Arain, A. A., Rizvi, W. H., & Vighio, M. S. (2021). Development and validation of user experience-based e-learning acceptance model for sustainable higher education. *Sustainability*, *13*(11). doi:10.3390/su13116201

Zhao, L., Zhao, B., & Li, C. (2023). Alignment analysis of teaching–learning-assessment within the classroom: how teachers implement project-based learning under the curriculum standards. *Disciplinary and Interdisciplinary Science Education Research*, *5*(1). doi:10.1186/s43031-023-00078-1