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Junior High School Students' Mathematical Thinking Ability on Set Theory Material Based on the APOS Theory

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Abstract

This study aimed to describe the mathematical thinking of seventh-grade students on set material and to interpret the dominant cognitive tendencies reflected in their responses through an APOS-informed perspective. This descriptive qualitative study involved 19 seventh-grade students at Srijaya Negara Junior High School in Palembang. Data were collected through a written test organized around four mathematical thinking indicators, namely specializing, generalizing, conjecturing, and convincing, as well as semi-structured interviews. In this study, APOS theory was used as an interpretive lens rather than as a strict one-to-one measurement framework. The results showed that, within the sampled group, students' average mathematical thinking score was in the good category (69.80). Indicator-level analysis revealed the highest achievement in specializing (81.9%) and the lowest in convincing (59.0%). Qualitative analysis of selected students from contrasting performance levels showed that the higher-performing student provided more integrated explanations, the middle-performing student relied more on procedural responses with partial justification, and the interviewed student in the less category showed difficulty in responding to the conjecture-and-proof task, although time limitation also affected task completion. These findings suggest that students tended to perform better in representational and procedural tasks than in justification-oriented tasks. The study implies that mathematics learning on set material should provide greater support for reasoning, explanation, and justification.

Keywords: APOS theory; junior high school students; mathematical thinking ability; set theory.

INTRODUCTION

Mathematical thinking is an essential skill for students to master because it underpins the solution of various problems (Monteleone et al., 2023). Mathematical thinking is a fundamental aspect of mathematics education and encompasses a series of cognitive processes (Ibrahim et al., 2023). Cognitively, mathematical thinking involves searching for, reasoning, analyzing, and interpreting information logically and accurately, enabling students to understand and apply mathematical concepts systematically (Van Doc et al., 2023). This ability is highly beneficial for students, both in mathematics learning and in dealing with everyday problems that require logical thinking (Irawati & Sofian Hadi, 2025). Through mathematical thinking, students can develop perspectives, ask questions, utilize relevant data, and convey ideas clearly and concisely (Danoebroto et al., 2024). Students skilled in mathematical thinking can meet indicators such as specializing, generalizing, conjecturing, and convincing, which involve analyzing problems, finding patterns, making conjectures, and proving conclusions (Delima et al., 2021; Dewi et al., 2024; Susanti et al., 2024). Thus, mathematical thinking is key to mastering mathematics, reasoning logically, and solving everyday problems.

Mathematical thinking skills are not only understood as internal, abstract mental processes, but can also be observed in students' concrete activities when solving mathematical problems. The way students represent problems, choose strategies, discover patterns, and formulate reasons and justifications for given solutions are manifestations of mathematical thinking processes that can be identified and measured. International studies indicate that reasoning, conjecturing, and constructing arguments are important indicators of students' mathematical thinking skills in tangible, measurable ways (Lithner, 2008; Niss & Højgaard, 2019; A. J. Stylianides, 2007). Thus, mathematical thinking skills can be studied through students' written responses and analyzed using a theoretical framework that represents the construction of students' mathematical knowledge.

However, previous studies indicate that students still experience difficulties in several aspects of mathematical thinking, particularly in reasoning and problem solving (Jawad & Majeed, 2021). The results of the Trends in International Mathematics and Science Study (TIMSS) show that Indonesian students' achievement remains below the international average, especially in reasoning and problem-solving domains (Mullis et al., 2015). In line with this, research by Jawad and Majeed (2021) reveals that students are still weak in logical thinking and experience difficulties in solving mathematical problems. In the context of set theory, students also encounter difficulties in solving mathematical problems, as reflected in low learning outcomes and errors in interpreting problem situations (Lestari et al., 2022; Pebriyanti & Amelia, 2023). Most students frequently make mistakes or errors when solving mathematical word problems on sets, as many misunderstand the underlying content of these word problems (Hasanudin & Habsyi, 2023). Furthermore, students experience difficulties in performing set operations, such as determining set elements, identifying universal sets and subsets, and translating contextual problems into mathematical models (Aulia & Kartini, 2021; Ismail et al., 2024). These findings suggest that difficulties in set material are not merely procedural, but are also related to how students represent, relate, generalize, and justify mathematical ideas. Therefore, a theoretical framework that can describe students' thinking processes in stages is needed to prevent these problems from occurring (Niss & Højgaard, 2019; G. J. Stylianides et al., 2024).

One cognitive theory that can describe students' thinking processes is APOS theory (Syaiful et al., 2025). APOS theory can be used to predict students' ability to understand and form mathematical concepts in their minds, and to help researchers or teachers explain in detail students' thought processes as reflected in written answers and interviews (Herawaty et al., 2020). This theory consists of four stages: action, process, object, and schema (Arnawa et al., 2021; Umam et al., 2025). In the action stage, students use mathematical concepts procedurally by following the teacher's instructions or examples. The second stage is the process, when students begin to understand the steps and can carry them out from their own thoughts without looking at examples. The third stage is the object stage, when students view the mastered process as a complete concept that can be used and manipulated to solve other problems. The final stage is schema, when students have a complete body of knowledge about mathematical concepts, enabling them to select and use appropriate concepts when faced with new or unfamiliar problems (Baye et al., 2021; Trigueros et al., 2024). APOS theory can also be used to enhance higher-order mathematical thinking skills (Mudrikah, 2016).

APOS theory is relevant to the study of mathematical thinking because it provides a structured way to interpret how students' understanding develops from procedural activity toward more connected conceptual reasoning (Dubinsky & McDonald, 2001; Martínez-Planell & Trigueros, 2019; Trigueros et al., 2024). In this study, APOS theory is not used as a rigid one-to-one measurement framework for the indicators of specializing, generalizing, conjecturing, and convincing. Instead, it is employed as an interpretive lens to describe the dominant cognitive tendencies reflected in students' responses. Through this perspective, students' responses can be interpreted in terms of whether they mainly reflect action-dominant, process-dominant, object-oriented, or schema-oriented reasoning.

Previous research shows that the APOS theory has been widely used to assess various students' mathematical abilities. Syaiful et al. (2025) used APOS theory in basic mathematics learning to improve students' mathematical critical thinking skills. Umam and Susandi (2022) identified students' critical thinking errors in solving mathematical problems by using APOS theory as a framework for analyzing students' mental stages. Furthermore, Yerizon et al. (2024) examined how students construct an understanding of the concept of partial derivatives through the APOS cognitive stages. At the junior high school level, Rahman et al. (2025) applied an APOS approach to analyze students' conceptual understanding in relations and functions and found that students tended to perform better on basic tasks than on more complex ones. However, studies specifically examining junior high school students' mathematical thinking in set material through the indicators of specializing, generalizing, conjecturing, and convincing, while interpreting students' responses through an APOS-informed perspective, remain limited. Thus, the gap addressed in this study lies not merely in applying APOS theory to another mathematical topic, but in using it to interpret the profile of students' mathematical thinking in set material, where representational, procedural, and justificatory demands appear in sequence.

The novelty of this study lies in two aspects. First, this study profiles junior high school students' mathematical thinking on set material using four explicit indicators, namely specializing, generalizing, conjecturing, and convincing. Second, this study interprets students' responses to those indicators through an APOS-informed perspective in order to describe the dominant cognitive tendencies underlying their solutions. This approach is expected to provide a more nuanced description of students' strengths and difficulties, especially in distinguishing between relatively strong representational or procedural performance and weaker justificatory reasoning.

Based on this background, the research questions of this study are as follows: (1) How is the profile of junior high school students' mathematical thinking on set material in terms of specializing, generalizing, conjecturing, and convincing? (2) How can students' responses to those indicators be interpreted through an APOS-informed perspective? (3) What characteristics are shown by selected students from different achievement categories in solving set problems? Therefore, this study aims to describe the profile of junior high school students' mathematical thinking on set material and to interpret the dominant cognitive tendencies reflected in their responses through an APOS-informed perspective.

METHOD

This study employed a descriptive qualitative design. The study was intended to describe the profile of junior high school students' mathematical thinking on set material and to interpret the dominant cognitive tendencies reflected in their responses through an APOS-informed perspective. A qualitative descriptive approach was considered appropriate because the study focused not only on students' final answers, but also on how they represented problems, explained procedures, formulated conjectures, and justified their conclusions through written work and interviews. The use of written tests and interviews in APOS-informed mathematics education research has also been widely employed to examine students' thinking processes in depth (Martínez-Planell & Trigueros, 2019; Sopamena et al., 2021).

This research was conducted in November 2025 at Srijaya Negara Junior High School, Palembang City, South Sumatra Province. The participants were all 19 students of one seventh-grade class, and total sampling was used because the study involved the entire class. Based on the written test results, three students were purposively selected for interviews to provide contrasting cases of higher, middle, and lower performance. These interview data were used to deepen the interpretation of students' written responses and were treated as case-based descriptions rather than as full representations of all students in each achievement category.

The instruments used in this study consisted of a written test, an analytical scoring rubric, and a semi-structured interview guide. The written test comprised three problem sets on set material, divided into 12 scorable components with a maximum total score of 39. The scoring was organized around four mathematical thinking indicators, namely specializing, generalizing, conjecturing, and convincing. The tasks were designed to elicit students' abilities in representing set situations, carrying out set operations, identifying patterns, formulating conjectures, and providing mathematical justifications. The analytical rubric used a score range of 0–3 for each component, with higher scores indicating more accurate, complete, and logically justified responses. To maintain clarity in the main text, the manuscript presents only a summary of the test blueprint and scoring structure.

In this study, APOS theory was not applied as a strict one-to-one scoring framework for the indicators of specializing, generalizing, conjecturing, and convincing. Instead, APOS theory was used as an interpretive lens to describe the dominant cognitive tendency reflected in students' responses. In this perspective, action-dominant responses referred to direct identification and representation of given information, process-dominant responses referred to procedural explanation and recognition of relationships, object-oriented responses referred to forming conjectures or general relationships, and schema-oriented responses referred to coherent justification or proof. Thus, the four mathematical thinking indicators remained the primary basis for scoring, while APOS theory functioned as an analytical perspective for interpreting the cognitive characteristics of students' work.

Before being used in the main study, the instruments underwent expert validation and limited pilot testing. The validation process involved the lecturer responsible for the mathematical thinking course and a mathematics teacher at Srijaya Negara Junior High School. The review focused on the suitability of the content with the topic of sets, the relevance of the items to the intended indicators, the clarity of the wording, and the appropriateness of the scoring rubric and interview prompts. Revisions were made based on the suggestions obtained during this stage, and the pilot testing was conducted to ensure that the questions could be understood by students and that the scoring criteria could be applied consistently.

Data collection was carried out in three stages: preparation, written testing, and interviewing. In the preparation stage, the researchers discussed the research plan with the supervising lecturer, communicated with the school and mathematics teacher, and revised the instruments based on expert input. In the implementation stage, the written test was administered to all 19 students according to the schedule agreed upon with the school. Students' responses were then scored using the analytical rubric. The total score obtained by each student was converted into a percentage using the formula:

$$\text{Score} = \left(\frac{\text{total score obtained}}{39}\right) \times 100$$

The obtained scores were then categorized into five levels of mathematical thinking ability: excellent (81–100), good (61–80), sufficient (41–60), less (21–40), and significantly less (0–20). This categorization follows the principle of converting scores into qualitative values proposed by Arikunto (2009), with interval adjustments tailored to research needs.

Table 1. Categories of Students' Mathematical Thinking Ability Scores

Grade	Category
81 - 100	Excellent
61 - 80	Good
41 - 60	Sufficient
21 - 40	Less
0 - 20	Significantly Less

Data analysis was conducted descriptively through quantitative and qualitative steps. First, students' written responses were scored across the 12 scorable components, then converted into percentages and grouped into the predetermined ability categories. In addition to total scores, students' performances were summarized by indicator to identify patterns of strength and difficulty across the class. Second, qualitative analysis was conducted on the written responses and interview transcripts of the selected students. This analysis was intended to describe the characteristics of students' responses at contrasting achievement levels and to interpret the dominant cognitive tendencies reflected in those responses through an APOS-informed perspective. The interpretation did not rely solely on aggregate scores, but also considered indicator-specific responses, especially those related to conjecturing and convincing, as well as students' explanations during interviews.

To maintain the trustworthiness of the data, several strategies were used. First, the instruments were reviewed by experts and revised before use. Second, pilot testing was conducted to improve the clarity and usability of the items and scoring criteria. Third, technical triangulation was applied by comparing data from written tests and interviews. The interview data were used to confirm, clarify, and strengthen the interpretation of students' written responses, particularly in cases where written work alone did not fully reveal students' reasoning processes.

RESULTS

This study involved 19 seventh-grade students at Srijaya Negara Junior High School. The results of the mathematical thinking ability test analysis showed that students' average score was 69.80. As shown in Table 2, students' mathematical thinking ability was generally in the good category, with the majority in the good (52.63%) and excellent (26.32%) categories, while a small portion was in the sufficient and less categories. There were no students in the significantly less category.

Table 2. Distribution of Students Based on Mathematical Thinking Ability Categories

Category	Score Range	Number of Students	Percentage of Success
Excellent	81 - 100	5	26,32%
Good	61 - 80	10	52,63%
Sufficient	41 - 60	3	15,79%
Less	21 - 40	1	5,26%
Significantly Less	0 - 20	0	0%

Further analysis was conducted based on the four mathematical thinking indicators, namely specializing, generalizing, conjecturing, and convincing, which were interpreted through an APOS-informed perspective. A summary of the average percentage achievement for each indicator is presented in Table 3.

Table 3. Average percentage of indicator achievement

Mathematical Thinking Indicator	APOS-Informed Interpretation	Average Percentage	Category
Specializing	Action-dominant	81,90%	Excellent
Generalizing	Process-dominant	70,20%	Good
Conjecturing	Object-oriented	70,50%	Good
Convincing	Schema-oriented	59,00%	Sufficient

APOS theory was used as an interpretive lens to describe dominant cognitive tendencies rather than as a strict one-to-one measurement framework. The data in Table 3 show clear descriptive differences across indicators. The specializing indicator obtained the highest average percentage, namely 81.9%, and was categorized as excellent. This result indicates that most students were able to identify the information given in the problem, represent it appropriately, and perform the initial steps of solution procedures. The generalizing and conjecturing indicators obtained 70.2% and 70.5%, respectively, and both were categorized as good. These results indicate that many students were able to identify patterns and use previously learned procedures when solving similar set problems. In contrast, the convincing indicator obtained the lowest average percentage, namely 59.0%, and was the only indicator in the sufficient category. A more specific result appeared in Question 3, which was designed to elicit justification and proof, where the convincing-related achievement reached only 43.80%. This noticeable gap between the specializing indicator (81.9%) and the convincing indicator (59.0%) suggests that students tended to perform better on representational and procedural tasks than on tasks requiring mathematical justification.

Based on the indicator-level results, the qualitative analysis focused on Question 3 because this problem most clearly elicited students' abilities to explain, generalize, and justify set relationships within one sequence of tasks. In this question, students were required not only to determine the results of set operations, but also to formulate a general symbolic relationship and explain why the relationship was valid. Therefore, Question 3 provided richer evidence for describing differences in students' mathematical thinking across contrasting performance levels.

Higher-Performance Student

Figure 1 illustrates a more integrated response, in which the higher-performing student connected the set formula with the Venn diagram and gave a logical explanation for the result.

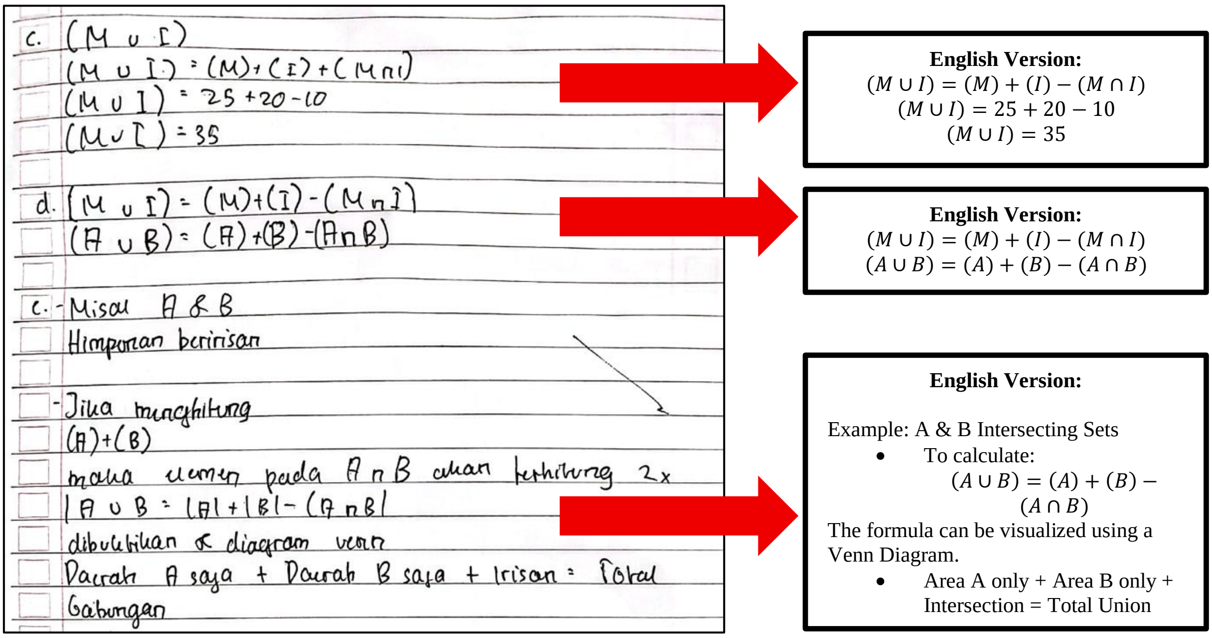

Figure 1. Higher-Performing Student's Answer to Question 3

The interview results show that the student understood the concepts used and was able to re-explain the solution steps by linking the set definition and its representation in Figure 1. The following is a transcript of the interview between the teacher (T) and a high-level student (S).

T : "How did you determine the formula for the relationship between the given sets in question number three, and what is your reasoning behind it?"

S : "Because the information in the problem states that M (Mathematics) has 25 students who like it, and I (Science) have 20 students who like it. Therefore, we can find the union of the sets that like Mathematics and Science by subtracting the students who like both subjects (the intersection). So, the formula can be written as

$$|M \cup I| = |M| + |I| - |M \cap I|.$$

This gives us the total number of students who like both subjects as 35.

T : "What is the meaning of the answer in question 3d, and why suddenly there are set A and set B?"

S : "It means that if there is a problem or question similar to number 3d, we can use that formula. For example, we can take set A and set B as examples, so the formula for finding the union would be $|A \cup B| = |A| + |B| - |A \cap B|$"

T : "Why does your symbolic proof answer convince others?"

S : "Because it is known that the total number of students in the class is 40, but if we directly calculate the union, where 25 students like Mathematics and 20 students like Science, the total comes to 45 students. Clearly, this is not relevant because $40 \neq 45$. Therefore, we must subtract the intersection of those who like both subjects to ensure the result matches the data."

T : "How does this formula connect with other set concepts, like the Venn diagram, as in your answer?"

S : "Before drawing the Venn diagram, we must first determine how many members are in each set and also the intersection. From the answer I have provided, it can be concluded that to find the union using the Venn diagram, you look at: 'The area of A only + the area of B only + the intersection = the total union.'"

The interviewed student representing higher performance was able to solve Question 3 correctly and provide a relatively coherent explanation. In the written response, the student used a Venn diagram appropriately and related it to the definitions of set membership, intersection, and union. The student also explained why the resulting formula matched the information given in the problem. For example, the student stated that if the values 25 and 20 were directly added, the total would become 45, which was inconsistent with the class size of 40; therefore, the intersection had to be subtracted once. This explanation indicates that the student was not only able to apply the union formula procedurally, but also to justify it by connecting numerical reasoning and diagrammatic representation. In APOS-informed terms, this response showed evidence of object-oriented reasoning supported by schema-oriented justification.

Middle-Performance Student

Figure 2 illustrates that the middle-performing student relied on an appropriate procedure, but the reasoning used to justify the result was still incomplete.

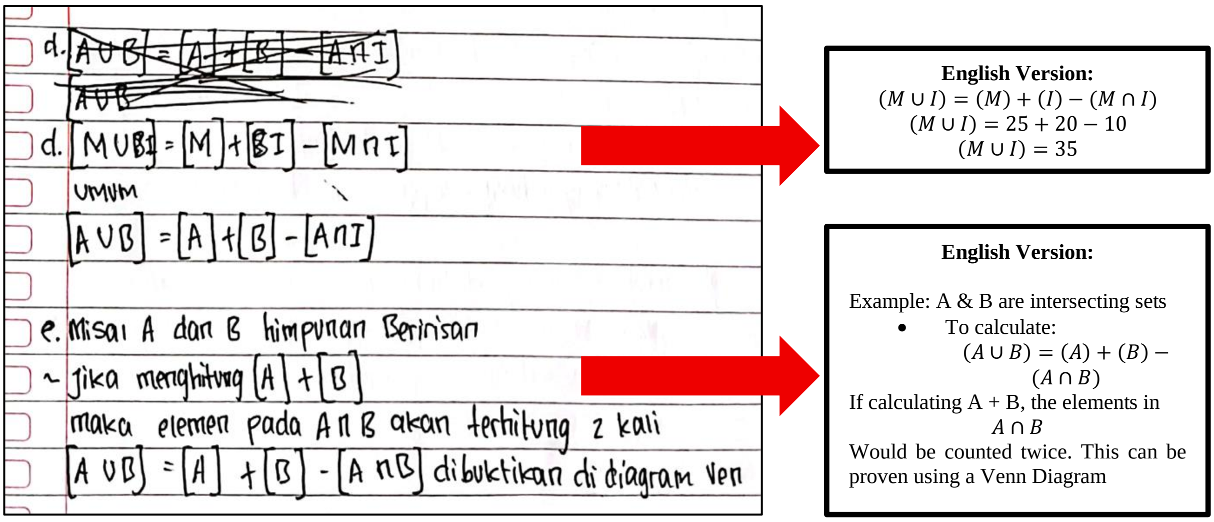

Figure 2. Middle-Performing Student's Answer to Question 3

According to interviews, students were able to explain the general steps of the process but struggled when asked to explain the reasoning or conceptual relationships underlying these steps. The following is a transcript of an interview between a teacher (T) and a student with an average ability (S).

T : "How did you determine the formula for the relationship between the given sets in question number three, and what is your reasoning behind it?"

S : "Because according to what we have learned, the formula for the union of Mathematics (M) and Science (I) is M plus I, then subtract the intersection. So, $|M \cup I| = 25 + 20 -$

10, the result is 35 students. I answered with this formula because it is the same as the one in the previous textbook."

T : "What is the meaning of the answer in question 3d, and why are set A and set B introduced?"

S : "I think A and B appear because if the problem is similar to this one, then these two sets are just an example, a more general one."

T : "Why does your symbolic proof convince others?"

S : "Because it is the same as what we have learned (with a confused expression)."

T : "How does this formula connect with other set concepts, like in the Venn diagram, as in your answer?"

S : "Because if A and B are intersecting sets, when calculating their union, they need to be separated. If we just add them together, the intersection of A and B would be counted twice. So the formula that can be used for other set concepts is $|A \cup B| = |A| + |B| - |A \cap B|$."

The interviewed student representing middle performance was generally able to determine the final result of Question 3, but the explanation remained limited and relied heavily on previously learned formulas. The written response showed correct use of the union formula, and during the interview the student explained that the formula was used because it was similar to what had been learned before. The student was also able to state that the intersection should not be counted twice when combining two sets. However, the student had difficulty explaining the conceptual relationship underlying the formula in a more coherent and independent way. Thus, this response suggests process-dominant reasoning with some movement toward object-oriented thinking, but with limited justificatory depth.

Interviewed Student in the Less Category

The interviewed student in the less category did not provide a written response to Question 3. During the interview, the student stated that the conjecturing and proving parts, especially sub-items 3d and 3e, were difficult to understand. However, the student also explained that time limitation affected task completion, because the student had spent too much time on Questions 1 and 2 and therefore did not have enough time to answer Question 3. This result suggests that the absence of a written response may have been influenced by both conceptual difficulty and test-time limitation. Therefore, the available evidence does not support a firm conclusion about the student's cognitive tendency in relation to the higher-level reasoning demanded by Question 3. The following is a transcript of the interview between the teacher (T) and a student in the less category (S).

T : "In question number three, you didn't write anything. Were there any difficulties when solving that question?"

S : "I think that question was too difficult to understand, so I couldn't answer it."

T : "Can you give an example of what you found difficult?"

S : "Making guesses and proving the guesses (in questions 3d and 3e)."

T : "So, did you manage to solve questions 3a to 3c?"

S : "I think I could… (with an unsure expression)."

T : "How did you convince yourself that you could answer questions 3a–3c?"

S : "Because this question is similar to questions number 1 and 2, but because I ran out of time when working on questions 1 and 2, I didn't have time to complete questions 3a–3c. So, for question 3, I didn't write anything."

Overall, the qualitative findings are consistent with the indicator-level results. Students tended to perform more strongly on tasks involving representation and direct procedures, while greater difficulty appeared when they were required to formulate general relationships and justify them mathematically. Accordingly, the main pattern emerging from the results is not that all students had weak mathematical thinking overall, but that their performance varied across indicators, with the most substantial difficulty appearing in the convincing indicator.

DISCUSSION

The findings of this study show that junior high school students' mathematical thinking on set material was generally in the good category, as reflected in the average score of 69.80 and the dominance of the good and excellent categories. However, this overall result did not indicate equal performance across all indicators. The indicator-level analysis revealed that students performed strongest in specializing and weakest in convincing, while generalizing and conjecturing were in the good category. This pattern indicates that students were relatively successful in identifying information, representing set situations, and carrying out direct procedures, but they experienced greater difficulty when they were required to construct mathematical arguments and justify general relationships. Thus, the results of this study suggest that a good overall score does not necessarily imply equally strong performance across all dimensions of mathematical thinking.

In set theory, such tasks mainly involve reading data and translating it into Venn diagrams or simple set operations, which are more accessible because they can be performed directly from the given information. This result is consistent with Tiengyoo et al. (2024) findings, which state that secondary school students can complete basic set operations because the steps are algorithmic and can be directly followed. Similarly, Lapele et al. (2024) found that students more easily demonstrate responses associated with early cognitive construction because these tasks do not yet require the integration of multiple concepts. In APOS-informed terms, these responses can be interpreted as action-dominant, since students mainly worked by identifying and representing the given information before moving toward deeper reasoning. The action stage is the easiest in the APOS structure because students only need to manipulate objects according to instructions, without understanding the conceptual meaning in depth (Bintoro et al., 2021). These findings suggest that procedural dominance is a common phenomenon in junior high school mathematics learning.

The generalizing and conjecturing indicators were both in the good category, showing that many students were able to move beyond direct representation and begin identifying patterns or forming initial general relationships. However, the interview data suggest that this success was not always supported by stable conceptual understanding. The higher-performing student was able to understand all elements of the task and showed all indicators clearly, whereas the middle-performing student could answer the questions but left some parts incomplete, especially in the more demanding subparts of Question 3. This finding suggests that students with moderate performance were often able to use previously learned procedures, yet some parts of generalizing, conjecturing, and convincing did not appear consistently in their responses. This condition is in line with Sari (2024), who showed that students' ability to generalize concepts and make conjectures is influenced by their mathematical ability level, and with Yerizon et al. (2024), who reported that students may carry out procedures correctly but still have difficulty providing coherent conceptual explanations. Lithner (2008) also explained that many students rely on imitative reasoning, namely solving tasks by recalling familiar patterns rather than constructing deeper conceptual structures. Therefore, the good category achieved in these two indicators reflects progress toward more flexible reasoning, but it does not yet indicate consistently well-developed conceptual connections.

The convincing indicator obtained the lowest percentage, especially in Question 3, which required students to formulate and justify a general symbolic relationship involving the union of two sets. This indicator demanded more than procedural execution because students had to explain why the formula worked, connect numerical information with diagrammatic representation, and justify the symbolic relationship logically. The qualitative data reinforce this pattern. The higher-performing student was able to coordinate the numerical data, the Venn diagram, and the symbolic expression coherently. The middle-performing student could produce the formula but gave limited or incomplete justification. The interviewed student in the less category showed substantial difficulty in responding to the task, which appeared to be related not only to limited understanding of the material and formulas, but also to time limitation during the test. This pattern is consistent with Tatira (2021), who found that many students struggle to reach more integrated forms of reasoning when they must connect definitions, representations, and procedures. Sukirwan et al. (2020) also showed that students' mathematical justification abilities are generally weaker than their ability to obtain answers procedurally.

The contribution of this study does not lie merely in confirming that justification tasks are more difficult than procedural tasks, as this is already widely recognized in mathematics education. Rather, the APOS-informed analysis helps identify a more specific pattern in learning set theory. In this study, students were generally able to represent information and perform direct set operations, and many were also able to identify patterns or make initial conjectures. However, greater difficulty emerged when they were required to transform those results into a general symbolic relationship and explain why the relationship was valid. This means that the main challenge in set theory learning is not only performing operations such as intersection or union, but also coordinating representations, procedures, and symbolic reasoning into a coherent justification. In this sense, the APOS-informed perspective provides a more specific interpretation of where students' reasoning begins to weaken, namely in the transition from process-dominant and object-oriented responses toward schema-oriented justification. International research by Tsafe (2024) confirms that APOS-based learning is effective only when students gradually build conceptual relationships, with the schema stage being the most complex because it requires comprehensive coordination of actions, processes, and objects. A similar finding was reported in a systematic literature review by Soku et al. (2025), which stated that the most significant obstacle in APOS-based research over the past two decades has been students' difficulty reaching the schema stage. This is the point at which students are no longer only solving a task, but also explaining and defending the validity of the mathematical relationship they use.

This interpretation is also supported by previous APOS-related studies. Students generally find it easier to demonstrate responses associated with action and process than those requiring more integrated reasoning (Sopamena et al., 2021; Trigueros et al., 2024). However, the present study adds a more specific insight in the context of set theory: the difficulty becomes more visible when students move from case-based reasoning to general symbolic reasoning. In Question 3, students first worked with concrete numerical information, then with the union result, and finally with a symbolic generalization and proof. This sequence makes visible a cognitive transition that may remain hidden if student performance is judged only from correct final answers. Thus, the APOS-informed analysis in this study contributes not only by describing which indicator was lowest, but also by showing how the difficulty emerged across a sequence of representational, procedural, generalizing, and justificatory demands within one topic.

The qualitative findings further strengthen this interpretation. The higher-performing student demonstrated a more complete form of mathematical thinking by understanding all elements of the problem and producing all indicators clearly. The middle-performing student could understand and answer most tasks, but some important parts were omitted, especially in the more complex and tricky parts of Question 3. This suggests that even students with fairly good scores may still have incomplete reasoning patterns when tasks demand more than direct calculation. Meanwhile, the interviewed student in the less category should be interpreted cautiously: the student appeared confused during the interview and had difficulty understanding the problem, the material, and the formula, but the student also reported that time affected performance. Therefore, this case should not be interpreted solely as evidence of a particular cognitive stage, but rather as a combination of conceptual difficulty and task-completion constraint. This more cautious reading is important so that the qualitative findings remain consistent with the available evidence.

The findings of this study have important implications for mathematics learning in junior high school, especially in teaching set material. Learning should not stop at identifying set elements, drawing Venn diagrams, or applying formulas mechanically. Students also need structured opportunities to explain why a formula is valid, compare numerical and visual representations, and move from specific cases to general symbolic relationships. Tsafe (2024) emphasizes that effective mathematics learning should support gradual transitions in students' conceptual development, while Sukirwan et al. (2020) suggest that teachers need to provide tasks that require students not only to calculate, but also to explain the reason behind each step. In the context of set theory, activities such as reflective questioning, classroom discussion, and conceptual exploration of Venn diagrams and symbolic relationships may help students strengthen their justification ability. Therefore, the instructional implication of this study is not simply to increase the number of exercises, but to design learning experiences that explicitly support the development of justification-oriented reasoning.

Overall, the results of this study indicate that mathematical thinking on set material should be understood as multidimensional. Students may perform well in representational and procedural tasks, yet still experience difficulty in forming coherent justifications. Accordingly, strengthening mathematical learning requires attention not only to the correctness of answers, but also to the processes by which students relate information, formulate general relationships, and defend the validity of their conclusions. In this way, learning can better support students' movement from direct procedural success toward more connected and meaningful mathematical reasoning.

CONCLUSION

This study shows that, within the sampled group of seventh-grade students, mathematical thinking on set material was generally in the good category, but the students' performance was not evenly distributed across all indicators. The students tended to show stronger performance in tasks involving identifying information, representing set situations, and carrying out direct procedures, while greater difficulty appeared in tasks requiring justification and proof. An APOS-informed interpretation suggests that many responses were adequately developed at the level of direct representation and procedural reasoning, but fewer responses demonstrated well-connected justification-oriented reasoning. Thus, the main issue identified in this study is not a uniformly low level of mathematical thinking, but an imbalance across indicators, especially between representational-procedural performance and justificatory reasoning.

The qualitative findings support this conclusion. The higher-performing student was able to understand the task comprehensively and demonstrated all indicators clearly. The middle-performing student could answer most tasks but still left some parts incomplete, especially in the more demanding sections of Question 3. Meanwhile, the interviewed student in the less category showed difficulty in understanding the problem, the material, and the relevant formulas, although time limitation also affected task completion. Therefore, this case should be understood as a specific observation within the sampled group rather than as a general characteristic of all lower-performing students. In this sense, the APOS-informed perspective contributes not merely by confirming that advanced reasoning is more difficult than basic procedures, but by showing that, in set theory tasks, the main difficulty becomes visible when students are required to move from case-based reasoning toward general symbolic justification.

This study is limited by its descriptive design, the small sample size of 19 students from one school, and the use of only three interview subjects as contrasting cases. Therefore, the findings should be interpreted as specific to the participants and context of this study and should not be generalized broadly to all junior high school students. Future research may build on these findings by designing and testing targeted instructional interventions that support students' movement from procedural and pattern-based responses toward stronger justification-oriented reasoning, particularly in set theory learning. Longitudinal studies and studies involving larger and more diverse samples are also needed to examine how students' mathematical thinking develops across APOS-informed cognitive tendencies over time.

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