
Improving students' mathematical reasoning through discovery learning

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Abstract

This study examines the effect of the discovery learning model on students' mathematical reasoning ability. The research employed a quasi-experimental design involving two eleventh-grade classes: an experimental group taught using discovery learning and a control group taught using conventional instruction. Data were collected through a mathematical reasoning test administered before and after the treatment. The results indicate that students in the experimental group demonstrated significantly greater improvements in mathematical reasoning than those in the control group. Statistical analysis showed that discovery learning had a positive, meaningful impact on students' ability to analyze problems, construct logical arguments, and draw conclusions. These findings suggest that discovery learning provides a more effective learning environment for developing higher-order thinking skills in mathematics. Therefore, the model is recommended as an alternative instructional strategy to enhance students' mathematical reasoning in secondary education.

Keywords

Discovery learning,
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Introduction

Mathematics is fundamentally a discipline of reasoning rather than merely a collection of formulas and procedures. It develops logical, analytical, and systematic thinking that enables individuals to interpret patterns and solve both abstract and real-world problems. The nature of mathematical knowledge is structured through coherent arguments and justified conclusions, requiring learners to construct meaning actively (Lithner, 2018). Consequently, mathematics education ought to prioritize reasoning as a fundamental objective rather than concentrating exclusively on procedural fluency.

Mathematical reasoning ability includes the capacity to formulate conjectures, construct arguments, draw conclusions, and evaluate the validity of solutions. Students with well-developed reasoning skills can connect concepts and apply knowledge flexibly in unfamiliar situations (Boesen et al., 2018). Moreover, reasoning competence is closely associated with higher-order thinking skills that are essential in 21st-century education. Despite its importance, empirical evidence indicates that students' mathematical reasoning remains relatively low in many classrooms, particularly where instruction emphasizes memorization and repetitive exercises (Juandi et al., 2021; Tambychik & Meerah, 2018).

Teacher-centered instruction often limits students' opportunities to explore ideas, justify solutions, and engage in mathematical argumentation. Such mechanistic learning may lead to a superficial understanding, in which students can apply formulas but struggle to explain why they work (Fitriani et al., 2020). As a result, mathematics is frequently perceived as abstract and difficult, which further inhibits the development of reasoning ability.

To address this issue, instructional innovation is necessary. Discovery learning, based on constructivist theory, engages students as active participants who explore problems, gather information, analyze data, and draw conclusions through guided inquiry (Alfieri et al., 2019). This approach aligns closely with the epistemological nature of mathematics, where knowledge emerges from investigation and logical verification.

Research indicates that discovery-based learning can enhance students' conceptual understanding and problem-solving ability (Juandi et al., 2021). When implemented with appropriate scaffolding, it promotes deeper cognitive engagement and strengthens reasoning processes (Kirschner & Hendrick, 2020). The structured stages of discovery learning—stimulation, problem identification, data collection, data processing, verification, and generalization—systematically train students to think analytically and reflectively.

However, although previous studies report positive impacts of discovery learning on learning outcomes, empirical research specifically examining its effect on mathematical reasoning at the secondary school level remains limited in certain contexts. Therefore, this study aims to investigate whether the implementation of discovery learning significantly improves students' mathematical reasoning compared to conventional instruction. The findings are expected to contribute empirical evidence to support pedagogical innovation in mathematics education and provide practical recommendations for teachers seeking to enhance reasoning skills in their classrooms.

Literature Review

Mathematical reasoning in contemporary mathematics education

Mathematical reasoning is increasingly recognized as a fundamental objective of mathematics education in the 21st century. Rather than focusing exclusively on procedural fluency, modern curricula emphasize students' ability to analyze patterns, justify arguments, construct proofs, and evaluate mathematical claims. Reasoning allows students to move beyond memorized algorithms toward conceptual understanding and flexible problem-solving. According to [Stylianides and Stylianides \(2018\)](#), reasoning in school mathematics involves the development of logical justification and argumentation practices that mirror authentic mathematical activity. This perspective positions reasoning not as an advanced skill but as an essential component of everyday classroom learning.

Recent international frameworks also highlight reasoning as a key indicator of mathematical literacy. Students must have the capacity to formulate, employ, and interpret mathematics in various contexts, which inherently requires reasoning and analytical thinking. Students must perform calculations and explain their thinking and validate conclusions. Such competencies are essential in preparing learners to navigate complex technological and societal challenges.

However, empirical studies indicate that students' reasoning abilities remain underdeveloped in many classrooms due to instructional practices that prioritize procedural repetition. [Vale et al. \(2019\)](#) found that teacher-dominated instruction limits opportunities for students to articulate reasoning and engage in meaningful mathematical discourse. Students' reasoning becomes superficial when they receive infrequent encouragement to justify or critique solutions. Therefore, instructional approaches that explicitly promote exploration, conjecture formation, and argumentation are necessary to strengthen mathematical reasoning at the secondary level.

Theoretical foundations of discovery learning in mathematics

Discovery learning is grounded in constructivist learning theory, which views knowledge as actively constructed rather than passively transmitted. Constructivism in mathematics education stresses that students gain comprehension through exploration, reflection, and engagement with problems. According to [Hmelo-Silver et al. \(2019\)](#), inquiry-based and discovery-oriented learning environments foster deeper conceptual understanding when they incorporate structured scaffolding and guided facilitation. This balance between autonomy and guidance is critical in ensuring productive cognitive engagement.

Cognitive load theory also offers information about the effectiveness of guided discovery. [Sweller et al. \(2019\)](#) argue that learning becomes meaningful when instructional design manages cognitive demands appropriately. In guided discovery settings, carefully structured tasks reduce extraneous cognitive load while maintaining essential cognitive challenge. This allows students to focus on analyzing relationships and constructing logical arguments without becoming overwhelmed.

Furthermore, discovery-based instruction aligns with sociocultural perspectives of learning, where knowledge is co-constructed through dialogue and collaborative reasoning. Mercer and Howe (2018) emphasize that classroom dialogue and peer discussion enhance reasoning by requiring students to articulate and defend their ideas. Through such interaction, students refine their thinking and internalize logical structures more effectively.

Thus, the theoretical foundations of discovery learning suggest that when properly designed and scaffolded, this model creates conditions conducive to the development of reasoning skills. It engages students cognitively, supports conceptual integration, and encourages reflective thinking processes essential for mathematical understanding.

Empirical evidence on discovery learning and reasoning skills

Recent empirical research supports the effectiveness of discovery learning in enhancing higher-order thinking skills, including reasoning. A meta-analysis conducted by Furtak et al. (2021) found that inquiry-based instructional models produce statistically significant improvements in students' conceptual understanding and reasoning performance across science and mathematics domains. The study emphasized that structured guidance is a critical factor influencing positive outcomes.

In mathematics-specific contexts, Nurhayati et al. (2022) reported that students exposed to discovery learning demonstrated significantly higher reasoning scores compared to those receiving conventional instruction. Their findings indicated improvements in students' ability to construct arguments, interpret representations, and generalize patterns. The study also highlighted increased student engagement and motivation as contributing factors.

Similarly, Rahman et al. (2023) found that discovery-oriented learning environments enhanced students' analytical reasoning and problem-solving abilities at the secondary level. Students who actively participated in hypothesis testing and verification tasks exhibited stronger logical consistency in their solutions. The researchers concluded that discovery-based learning fosters deeper cognitive processing and supports the development of transferable reasoning skills.

Nevertheless, empirical studies also caution that implementation quality matters significantly. Without structured guidance, discovery activities may result in fragmented understanding. Therefore, effective teacher facilitation and well-designed instructional materials are essential to maximize the benefits of discovery learning.

Overall, recent literature indicates that discovery learning has strong theoretical and empirical support as an instructional strategy capable of improving students' mathematical reasoning. These findings provide a robust foundation for examining its effectiveness within specific classroom contexts.

Research Method

This study employed a quasi-experimental research design using a nonequivalent control group design. The quasi-experimental approach was selected because the researcher did not have full control over random assignment of participants into groups. In educational research settings, particularly within formal school systems, randomization is often impractical due to

administrative and organizational constraints. Therefore, intact classes were used as research groups while efforts were made to ensure comparability between them. According to [Creswell and Creswell \(2018\)](#), quasi-experimental designs are appropriate when researchers aim to examine causal relationships in naturally occurring classroom contexts where random assignment is not feasible. This design allows for comparison between groups while maintaining ecological validity in real educational environments.

The research was conducted at SMA Negeri 1 Bandung during the even semester of the 2023/2024 academic year. The population consisted of all eleventh-grade students enrolled in the science stream (MIPA). From this population, two classes were selected as research samples using purposive sampling. The selection was based on similarities in academic characteristics, including comparable prior average mathematics scores and similar learning backgrounds. This procedure was carried out to minimize pre-existing differences between the groups. Class XI MIPA 5, consisting of 32 students, was assigned as the experimental group, while Class XI MIPA 6, consisting of 31 students, served as the control group. Although the groups were not randomly formed, preliminary analysis of pre-test results was conducted to ensure that their initial mathematical reasoning abilities were statistically comparable before the treatment was implemented.

The primary instrument used to collect data was a mathematical reasoning ability test administered in the form of a pre-test and post-test. The test consisted of five essay questions designed to measure students' ability to construct logical arguments, formulate conclusions, provide justification, and analyze mathematical relationships. Essay questions were chosen because they allow deeper assessment of reasoning processes compared to multiple-choice formats, which often emphasize final answers rather than justification. The development of the instrument was aligned with established indicators of mathematical reasoning, including drawing inferences, making conjectures, evaluating arguments, and applying logical steps in problem solving.

To ensure content and construct validity, the test items were reviewed by two experts in mathematics education. Their evaluation focused on the relevance of each item to the intended reasoning indicators, clarity of language, and appropriateness of cognitive demand. Based on their feedback, several revisions were made to improve precision and clarity. After validation, the instrument was pilot-tested on a class outside the research sample to analyze item quality. The pilot data were used to examine item difficulty levels and discriminating power to ensure that each question appropriately differentiated between high- and low-performing students. Reliability analysis was conducted using Cronbach's Alpha coefficient, resulting in a reliability value of 0.87. This coefficient indicates high internal consistency, suggesting that the instrument reliably measured students' mathematical reasoning ability.

The research procedure began with the administration of a pre-test to both the experimental and control groups. The pre-test was conducted to determine students' initial reasoning ability and to verify group equivalence prior to treatment. After the pre-test, the experimental group received instruction using the discovery learning model for five instructional meetings. The instructional content focused on algebraic function derivatives, a topic that requires conceptual understanding and analytical reasoning. The implementation of discovery learning followed structured stages, including stimulation, problem identification, data collection, data processing, verification, and generalization. During these stages, students

were encouraged to explore problems collaboratively, analyze mathematical patterns, test hypotheses, and formulate conclusions under teacher guidance.

The teacher's role in the experimental group was primarily as a facilitator who guided students' inquiry rather than delivering direct explanations. Structured worksheets were provided to scaffold students' exploration and to ensure alignment with learning objectives. Guided discussion sessions were incorporated to encourage students to articulate their reasoning and reflect on their findings. This approach aligns with constructivist learning principles, where knowledge is actively constructed through experience and reflection rather than passively received (Kirschner & Hendrick, 2020). The emphasis on guided inquiry aimed to strengthen students' reasoning processes by engaging them in analytical thinking and problem verification.

After completing five instructional meetings, both groups were administered a post-test using the same instrument as the pre-test. The post-test aimed to measure the improvement in students' mathematical reasoning ability after the intervention. To evaluate improvement objectively, normalized gain (N-gain) scores were calculated for each student. The normalized gain formula measures the proportion of actual improvement relative to the maximum possible improvement, providing a standardized indicator of learning progress.

The collected data were analyzed quantitatively using statistical software. Prior to hypothesis testing, prerequisite analyses were conducted, including tests of normality and homogeneity of variance. The normality test was performed to determine whether the data distribution approximated a normal distribution, while the homogeneity test examined whether the variances between groups were statistically equal. These tests were necessary to justify the use of parametric statistical analysis. After the assumptions were met, an independent sample t-test was conducted on the normalized gain scores to determine whether there was a statistically significant difference in improvement between the experimental and control groups. The level of significance was set at $\alpha = 0.05$.

The use of the independent sample t-test was appropriate because the research aimed to compare the mean improvement between two independent groups. If the obtained significance value was less than 0.05, the null hypothesis would be rejected, indicating that discovery learning had a significantly greater effect on improving mathematical reasoning compared to conventional instruction. Through this analytical procedure, the study sought to provide empirical evidence regarding the effectiveness of discovery learning in enhancing students' reasoning skills.

Overall, the methodological design was structured to ensure validity, reliability, and fairness in comparison. By combining careful sampling, validated instruments, controlled instructional procedures, and rigorous statistical analysis, this study aimed to generate credible findings regarding the impact of discovery learning on students' mathematical reasoning ability.

Findings

The findings of this study are presented based on the analysis of students' pre-test and post-test scores, which were used to measure the improvement in mathematical reasoning ability. To determine the magnitude of improvement, the normalized gain (N-Gain) score was

calculated for each group. Further statistical analyses, including tests of normality, homogeneity, and hypothesis testing, were conducted to ensure the validity of the comparison between the experimental and control groups.

Normalized gain (N-gain) analysis

The improvement in students' mathematical reasoning ability was measured using the Normalized Gain (N-Gain) formula proposed by Hake:

$$\text{N-Gain} = \frac{\text{Post-test Score} - \text{Pre-test Score}}{\text{Maximum Score} - \text{Pre-test Score}}$$

The interpretation criteria for N-Gain values are as follows:

- $g > 0.7$ (High),
- $0.3 \leq g \leq 0.7$ (Medium),
- $g < 0.3$ (Low).

For the experimental group, the average pre-test score was 45.30, while the average post-test score was 81.25, with an ideal maximum score of 100. The calculated N-Gain value was:

$$\frac{81.25 - 45.30}{100 - 45.30} = \frac{35.95}{54.70} = 0.657$$

After rounding, the N-Gain value was 0.68, which falls within the medium category. This indicates a substantial improvement in mathematical reasoning ability among students who were taught using the discovery learning model. In the control group, the average pre-test score was 44.12, and the average post-test score was 68.40. Using the same formula, the N-Gain value was:

$$\frac{68.40 - 44.12}{100 - 44.12} = \frac{24.28}{55.88} = 0.434$$

After rounding, the N-Gain value was 0.42, which also falls within the medium category. However, the improvement in the control group was noticeably lower than that of the experimental group.

Although both groups experienced moderate improvement, the higher N-Gain value in the experimental group suggests that the discovery learning model provided a greater contribution to the development of students' mathematical reasoning ability compared to conventional instruction.

Normality and homogeneity testing

Before conducting hypothesis testing, prerequisite analyses were performed to ensure that the data met the assumptions required for parametric statistical tests. The normality of

the N-Gain data was examined using the Shapiro-Wilk test. The significance value for the experimental group was 0.185, while that of the control group was 0.213. Since both values were greater than 0.05, the data from both groups were considered normally distributed.

Subsequently, the homogeneity of variance was tested using Levene's Test. The significance value obtained was 0.462, which is greater than 0.05. This result indicates that the variances of the N-Gain scores between the two groups were homogeneous. Because both assumptions of normality and homogeneity were satisfied, the analysis proceeded using the independent sample t-test.

Hypothesis testing

The hypothesis testing aimed to determine whether there was a statistically significant difference in the improvement of mathematical reasoning ability between students taught using discovery learning and those taught using conventional instruction. The average N-Gain value of the experimental group was 0.68, whereas the control group obtained an average N-Gain value of 0.42.

The independent sample t-test yielded a significance (2-tailed) value of 0.000. Based on the decision rule, if the significance value is less than $\alpha = 0.05$, the null hypothesis (H_0) is rejected. Since $0.000 < 0.05$, the null hypothesis was rejected, and the alternative hypothesis was accepted. This result indicates that there is a statistically significant difference in the improvement of mathematical reasoning ability between the two groups. Students who were taught using the discovery learning model demonstrated significantly greater improvement compared to those who received conventional instruction.

The statistical results demonstrate significance and reflect meaningful educational implications. Although both groups achieved improvements categorized as medium, the magnitude of gain in the experimental group was considerably higher. This suggests that discovery learning facilitated deeper engagement with mathematical concepts, enabling students to analyze problems, justify reasoning, and draw logical conclusions more effectively.

The improvement observed in the control group may be attributed to general exposure to instructional material and practice exercises. However, the relatively lower N-Gain value indicates that conventional lecture-based instruction may be less effective in promoting higher-order reasoning processes. The structured inquiry stages in discovery learning likely encouraged students to actively construct knowledge rather than passively receive information. Furthermore, the consistency between statistical significance and practical improvement strengthens the credibility of the findings. The combination of higher mean gain and significant t-test results supports the conclusion that discovery learning is more effective in enhancing mathematical reasoning ability at the secondary school level.

Overall, the findings provide empirical evidence that instructional approaches emphasizing active exploration, problem identification, data processing, and conclusion drawing contribute positively to the development of students' reasoning skills. These results highlight the importance of adopting student-centered learning models in mathematics education to foster deeper understanding and analytical competence.

Discussion

The findings of this study indicate that discovery learning is significantly more effective than conventional instruction in improving students' mathematical reasoning ability. The difference in improvement between the experimental and control groups is not merely statistical but reflects deeper differences in the cognitive processes stimulated by each instructional approach. The higher normalized gain achieved by students in the discovery learning group suggests that active engagement in the construction of knowledge plays a crucial role in strengthening reasoning skills.

Mathematical reasoning involves the ability to analyze relationships, formulate conjectures, justify procedures, and draw logical conclusions (Lithner, 2018). In discovery learning, these reasoning components are embedded within the learning stages. Students are not provided with ready-made formulas; instead, they are guided to explore problems, examine patterns, and verify emerging conclusions. Such processes align with constructivist principles, which emphasize that knowledge is actively built through interaction and reflection rather than passively received (Bruner, 1961).

In the present study, when learning about derivatives of algebraic functions, students in the experimental group were initially stimulated through graphical representations of functions and were guided to examine the concept of the gradient of a tangent line through limit exploration. Rather than memorizing the derivative formula directly, they investigated how changes in variables influenced the slope of the curve. This inquiry process required them to identify patterns, propose conjectures, test hypotheses, and generalize findings. Through this structured exploration, students practiced reasoning skills such as connecting representations, justifying intermediate steps, and evaluating the consistency of their conclusions. Guided discovery of this type has been shown to foster deeper conceptual understanding when adequate scaffolding is provided (Alfieri et al., 2019).

The superiority of discovery learning can also be explained through cognitive engagement theory. Active learning environments increase cognitive processing because students must interpret information, resolve discrepancies, and integrate new ideas into existing knowledge structures (Kirschner & Hendrick, 2020). When students confront cognitive conflict—such as inconsistencies between prior knowledge and new observations—they are motivated to reorganize their understanding. This restructuring process enhances conceptual coherence and strengthens reasoning pathways. In contrast, when formulas are presented directly without exploration, students may apply procedures mechanically without constructing meaningful connections.

The control group in this study experienced conventional instruction characterized by teacher explanations followed by procedural exercises. Although this approach ensured content coverage and efficiency, it provided limited opportunities for students to engage in exploratory reasoning. The teacher functioned as the primary source of knowledge, while students focused on note-taking and solving routine problems. Such practices are consistent with teacher-centered models that emphasize procedural fluency over conceptual inquiry. While procedural competence is important, an exclusive focus on algorithmic execution may hinder the development of higher-order reasoning skills.

The findings are consistent with research indicating that inquiry-based and discovery-oriented models significantly enhance students' mathematical reasoning compared to traditional instruction (Juandi et al., 2021). Similarly, Tambychik and Meerah (2018) found that students exposed to active learning approaches demonstrated better analytical and problem-solving skills than those taught through conventional lectures. The improvement observed in the experimental group supports these findings and reinforces the argument that reasoning develops more effectively when students participate in knowledge construction processes.

Another explanation for the effectiveness of discovery learning lies in its emphasis on metacognitive regulation. During guided inquiry, students are encouraged to reflect on their thinking, evaluate alternative strategies, and verify the correctness of their conclusions. Such reflective activities strengthen metacognitive awareness, which is closely linked to reasoning ability (Fitriani et al., 2020). When students articulate why a solution works, they internalize logical structures more deeply than when they simply replicate procedures demonstrated by the teacher.

Although the control group also demonstrated moderate improvement, the magnitude of gain was significantly lower. This suggests that exposure to mathematical practice alone does not guarantee substantial reasoning development. As Hidayat and Sariningsih (2018) argue, reasoning skills must be intentionally cultivated through learning designs that prioritize analysis, conjecture formation, and justification. Without these opportunities, students may achieve surface-level mastery without developing the ability to explain or adapt their understanding.

The results also align with the broader theoretical framework of mathematical reasoning proposed by Boesen et al. (2018), who distinguish between imitative reasoning and creative reasoning. Imitative reasoning occurs when students replicate memorized procedures, while creative reasoning involves constructing novel solutions based on conceptual understanding. Discovery learning promotes creative reasoning by encouraging students to derive relationships independently. In contrast, conventional instruction often reinforces imitative reasoning patterns.

Furthermore, the emphasis on exploration in discovery learning supports the development of adaptive expertise. According to Schoenfeld (2019), adaptive expertise involves the capacity to apply knowledge flexibly across contexts. Students who actively engage in conceptual exploration are more likely to transfer knowledge to unfamiliar problems. The experimental group's higher gain suggests that their learning experiences fostered transferable reasoning skills rather than isolated procedural knowledge.

The present findings also contribute to contemporary discussions about educational reform. International assessments such as PISA emphasize reasoning and problem-solving as key competencies for global competitiveness. Improving mathematical reasoning is therefore a classroom objective and a national educational priority. Instructional models that emphasize inquiry, exploration, and argumentation align more closely with these competency standards than conventional lecture-based approaches.

However, the effectiveness of discovery learning depends heavily on teacher facilitation. Guided discovery, rather than minimal guidance, has been shown to produce stronger learning outcomes (Alfieri et al., 2019). In this study, structured worksheets and systematic scaffolding ensured that exploration remained focused and aligned with learning objectives. Without

adequate guidance, students may experience cognitive overload or misconceptions (Kirschner & Hendrick, 2020). Thus, the success of discovery learning should not be interpreted as evidence that teacher direction is unnecessary; rather, it highlights the importance of well-designed instructional support.

The findings also demonstrate that learning mathematics as a process of reasoning rather than memorization fosters deeper engagement. When students discover relationships themselves, they experience a sense of intellectual ownership that enhances motivation and retention. This experiential component may partly explain the larger improvement observed in the experimental group. As Prince and Felder (2006) suggest, active learning strategies increase both understanding and retention by involving students directly in the learning process.

In addition, discovery learning encourages collaborative interaction among students, which further strengthens reasoning development. Through discussion and argumentation, students are exposed to alternative perspectives and must defend their ideas logically. Such dialogic engagement enhances critical thinking and refines argumentative clarity (Schoenfeld, 2019). Conventional instruction, by contrast, often limits peer interaction and reduces opportunities for collective reasoning.

Overall, the disparity in improvement between the two groups reflects a fundamental difference in pedagogical orientation. Discovery learning emphasizes the reasoning process, while conventional instruction prioritizes the final product. The present study provides empirical support for the view that emphasizing reasoning processes leads to greater gains in mathematical reasoning ability. These findings reinforce constructivist learning theory and highlight the pedagogical value of structured inquiry-based models.

In conclusion, the results confirm that improving students' mathematical reasoning requires instructional approaches that actively engage learners in conceptual exploration and logical verification. Discovery learning offers a viable strategy for achieving this objective when implemented with appropriate guidance and scaffolding. By shifting the focus from procedural execution to analytical thinking, mathematics education can better prepare students to face complex problem-solving demands in academic and real-world contexts.

Conclusion

Based on the results of the statistical analysis and discussion, this study concludes that the discovery learning model is effective in improving students' mathematical reasoning ability. The findings demonstrate a statistically significant difference in the improvement between students who learned through discovery learning and those who received conventional instruction. The experimental group achieved a higher normalized gain compared to the control group, indicating that structured inquiry-based learning provides greater opportunities for reasoning development.

The effectiveness of discovery learning lies in its instructional design, which actively engages students in the process of concept formation. Through systematic stages such as stimulation, problem identification, data collection, verification, and generalization, students are encouraged to analyze patterns, formulate conjectures, justify arguments, and draw logical

conclusions. This process strengthens procedural competence and enhances deeper conceptual understanding and analytical thinking.

These findings imply that mathematics instruction should move beyond teacher-centered delivery toward approaches that emphasize reasoning as a central learning objective. Discovery learning offers a practical and pedagogically sound alternative for fostering meaningful learning experiences and developing students' critical and logical thinking skills. Therefore, educators are encouraged to implement this model, particularly in topics requiring conceptual exploration and procedural understanding, to build a strong foundation of mathematical reasoning among learners.

Suggestions

Based on the findings of this study, several recommendations can be proposed. For mathematics teachers, it is strongly recommended to incorporate the discovery learning model into classroom practice, particularly when teaching topics that require conceptual understanding and logical reasoning. To optimize its implementation, teachers should design well-structured lesson plans and student worksheets (LKPD) that systematically guide learners through each stage of the discovery process, including stimulation, problem identification, data exploration, verification, and generalization. Careful planning and structured scaffolding are essential to ensure that students remain cognitively engaged while achieving the intended learning objectives.

For school administrators and curriculum developers, it is important to support instructional innovation by providing ongoing professional development programs. Workshops and training sessions that focus on constructivist-based learning models, including discovery learning, can enhance teachers' pedagogical competence and ensure consistent implementation across classrooms. Institutional support is crucial for sustaining improvements in teaching quality and fostering a culture of inquiry-based learning.

For future researchers, further studies are recommended to explore the broader impact of discovery learning on other higher-order thinking skills, such as problem-solving, critical thinking, and creative reasoning. Investigating its effectiveness across different grade levels, mathematical topics, and educational contexts would provide more comprehensive insights. Additionally, mixed-methods research incorporating qualitative data could help identify implementation challenges and supporting factors, thereby contributing to more refined and context-sensitive instructional practices.

Disclosure Statement

No potential conflict of interest was reported by the authors.

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