# STUDY OF SIMILARITY PATTERN OF CHLADNI PLATE VIBRATIONS OF CIRCULAR AND SQUARE GEOMETRY USING SSIM

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## ABSTRACT

*Chladni patterns are formed on plates excited at resonant frequencies and can be observed through the arrangement of particles on the plate surface. This study investigates the structural similarity between numerical and experimental Chladni patterns for circular and square plates using the Structural Similarity Index Measure (SSIM). Numerical vibration mode shapes for circular plates, namely modes (0,1) and (0,2), were obtained by solving the Helmholtz equation using Bessel functions in polar coordinates. Square plate modes (2,1) and (2,2) were modeled using Fourier cosine approximations to the biharmonic plate equation under free-edge boundary conditions. Experimental images were obtained from the Physics Demonstration Archive of the University of California, Santa Barbara, and were preprocessed to match the simulated patterns in terms of grayscale representation, resolution and contrast. Image comparison was performed using Python with the NumPy, SciPy, OpenCV and scikit-image libraries. The SSIM values for corresponding modes indicate a high level of similarity, reaching 0.9326 and 0.9103 for circular plates, and 0.9026 and 0.7517 for square plates, indicating strong structural agreement. Cross-mode comparisons produced significantly lower similarity values for circular plates, but relatively higher values for square plates due to shared Cartesian symmetry. The results demonstrate that SSIM is an effective metric for validating theoretical vibration models and quantifying modal similarity, while providing a practical image-based approach for Chladni pattern analysis.*

**Keywords:** *Chladni patterns; Image similarity; Resonance; SSIM; Vibration analysis*

## INTRODUCTION

Vibrations and waves constitute fundamental phenomena in physics, describing how mechanical energy propagates through solids, fluids and structures. These phenomena underlie a wide range of physical systems, from acoustic wave transmission and resonance in mechanical components to structural dynamics and material characterization. A classical and visually striking illustration of two-dimensional vibrational behaviour is provided by the pioneering work of Ernst Chladni, who demonstrated that fine particles sprinkled on thin metal plates arrange themselves into distinct geometric patterns when the plates are excited at specific resonant frequencies (Chladni, 2015). These patterns, now known as *Chladni figures*, form along nodal lines where transverse displacement is minimal, offering direct insight into the relationship between vibration modes, geometry and boundary conditions (Rossing & Fletcher, 2004; Jenny, 2001).

The vibrational behaviour of plates has long been analyzed using the Kirchhoff–Love plate theory, which leads to the biharmonic plate equation governing transverse motion in thin elastic plates (Leissa, 1969). For circular plates, the governing equations are naturally expressed in polar coordinates, yielding separable solutions in terms of Bessel functions. The roots of these functions determine the allowable vibration modes and corresponding nodal patterns, which manifest as concentric circles and radial nodal lines in Chladni figures (Handayani et al., 2018; Cianci et al., 2024). In contrast, square plates are formulated in Cartesian coordinates, where approximate solutions are commonly constructed using trigonometric or Fourier series. The resulting mode shapes exhibit grid-like, diagonal, or mixed nodal arrangements that are highly sensitive to boundary conditions, material properties, and geometric symmetry. Experimental studies have consistently shown strong correspondence between these theoretical predictions and observed nodal structures (D'Alessio, 2021; Kovacic & Kanovic, 2023).

Recent advances have significantly enhanced the experimental and computational investigation of vibrational patterns. High-resolution measurement techniques, such as laser Doppler vibrometers applied to conventionally manufactured and additively manufactured plates, have enabled precise identification of nodal lines and modal frequencies (Hagara et al., 2024). In parallel, open-source computational tools and Python-based frameworks for operational modal analysis have facilitated automated extraction and visualization of vibration modes from experimental data (Pasca et al., 2022; Rosso et al., 2023). These developments reflect a broader shift toward data driven and geometry-sensitive approaches in vibration analysis, extending classical analytical methods with computational and experimental validation.

Experimental studies of wave phenomena also play a crucial role in supporting theoretical models and physics education. Investigations into sound propagation, frequency variation, and source motion such as Doppler effect-based analyses in fluid environments, demonstrate how experimentally accessible measurements can validate wave theory and enhance conceptual understanding (Husni et al., 2024). Such studies highlight the importance of linking analytical formulations with experimentally observable patterns, particularly in educational and applied physics contexts where sophisticated measurement systems may be limited.

Alongside these experimental and computational advances, image-based analysis has emerged as a promising tool for quantitative comparison of vibrational patterns. The Structural Similarity Index Measure (SSIM), originally developed for image-quality assessment, evaluates similarity based on luminance, contrast, and structural information rather than pixel by pixel differences (Wang et al., 2004). Its application has expanded to pattern recognition, inspection and validation of modal shapes in structural dynamics and wave-based systems (Peng et al., 2020; Bakurov et al., 2022). For Chladni plates, SSIM offers a systematic and perceptually meaningful metric for comparing experimentally observed nodal patterns with numerically simulated mode shapes.

Despite these developments, relatively few studies have integrated analytical modelling, numerical simulation, and image-based similarity analysis within a single, unified framework for comparing Chladni patterns across different plate geometries. Existing research often focuses on either theoretical mode prediction or qualitative experimental observation, leaving a gap in quantitative validation methods that can directly assess structural correspondence between simulations and experiments.

Therefore, this study proposes a Python-based framework for generating and comparing Chladni patterns of circular and square plates using the Structural Similarity Index Measure as the primary metric of structural similarity. By combining wave phenomena on circular plate theory, numerical modelling and image-based similarity evaluation, this work provides a reproducible approach for validating theoretical vibration models against experimental data and deepens the understanding of geometry dependent modal behaviour in plate vibrations.

## THEORY

The vibration behaviour of circular and square Chladni plates in this study is described using the Kirchhoff–Love plate theory, which models the bending of thin elastic plates and leads to the biharmonic equation governing transverse displacement (Leissa, 1969). Analytical mode shapes derived for each plate geometry form the basis for numerical simulations of Chladni patterns. These simulated patterns are subsequently compared with experimental images using the Structural Similarity Index Measure (SSIM) to evaluate their structural correspondence.

### Circular Plate Analysis

For a circular plate of radius $a$ ,free transverse vibrations satisfy the Helmholtz equation in polar coordinates $(r, \theta)$:

$$\nabla^2 U + k^2 U = 0 \tag{1}$$

Assuming a separable solution:

$$U(r,\theta) = R(r)\Theta(\theta) \tag{2}$$

Substituting (2) into (1) separates the equation into radial and angular parts.

### Angular Component

$$\frac{\partial^2 \Theta}{\partial \theta^2} + m^2 \Theta = 0 \tag{3}$$

with solution:

$$\Theta(\theta) = A\cos(m\theta) + B\sin(m\theta) \tag{4}$$

Where $m = 0,1,2\ldots$ is an integer (due to periodicity in $\theta$), representing the number of nodal diameters.

### Radial Component

$$r^2 \frac{\partial^2 R}{\partial r^2} + r\frac{\partial R}{\partial r} + (k^2 r^2 - m^2)R = 0 \tag{5}$$

This is known as Bessel's equation and it has a general solution involving the first and second kind (Chakraverty, 2009). Since the plate that extends across the origin must have a finite displacement at $r$=0, the radial solution to the Helmholtz equation in polar coordinates is;

$$R(r) = CJ_m(kr) \quad (6)$$

The boundary condition $U(r = a, \theta) = 0$ gives $J_m(ka) = 0$, determines the permissible wave numbers defined as:

$$k_{mn} = \frac{j_{mn}}{a} \quad (7)$$

Thus, the full mode shape becomes:

$$U(r, \theta) = J_m(kr)[A_m \cos(m\,\theta) + B_m \sin(m\theta)] \quad (8)$$

and the corresponding natural frequencies are:

$$f_{mn} = \frac{\omega_{mn}}{2\pi} = \frac{j_{mn}c}{2\pi a} \quad (9)$$

where c is the modal wave speed, $f_{mn}$ is the natural frequency of the plate (in Hertz) for a specific vibrational mode defined by the indices $m$ and $n$, $j_{mn}$ is the $n^{th}$root of the Bessel function of the first kind $j_m(x)$. Circular plate modes exhibit characteristic radial nodal circles and diameters, forming the Chladni patterns used in this study.

### Square Plate Analysis

For a square plate of side length, *a*, the Kirchhoff Love plate equation in Cartesian coordinates is:

$$\frac{\partial^2 u}{\partial t^2} + \frac{D}{\rho l}\nabla^4 u \quad (10)$$

where, $u(x, y, t)$ is the transverse displacement, $D$ is Flexural rigidity, $\rho$ is the density of the plate, $l$ is the thickness of the plate and $\nabla^4$ is the biharmonic operator. The biharmonic operator in Cartesian coordinates, represents a fourth-order spatial derivative and is expressed as:

$$\nabla^4 = \frac{\partial^4 u}{\partial x^4} + 2\frac{\partial^4 u}{\partial x^2 \partial y^2} + \frac{\partial^4 u}{\partial y^4} \quad (11)$$

Assuming a separable solution:

$$u(x, y, t) = X(x)Y(y)T(t) \quad (12)$$

and substituting into (10) produces temporal and spatial equations.

### Temporal Equation

$$\frac{d^2 T}{dx^2} + \omega^2 T = 0 \quad (13)$$

with solution:

$$T(t) = A\cos(\omega t) + B\sin(\omega t) \quad (14)$$

### Spatial Equation

$$\nabla^4(XY) = \lambda(XY) \quad (15)$$

where $\lambda = \frac{\rho l \omega^2}{D}$

Under free or clamped boundary conditions, approximate mode shapes for square plates are commonly expressed using Fourier cosine series (D'Alessio, 2021):

$$u(x, y, t) = \sum_{m,n} A_{mn}\cos(m\pi x)\cos(n\pi y)\cos(\omega_{mn} t) \quad (16)$$

These mode produce grid-like or diagonal nodal lines characteristic of square-plate Chladni patterns (Kovacic & Kanovic, 2023).

### Structural Similarity Index Measure (SSIM)

The Structural Similarity Index Measure (SSIM) is employed in this study as a quantitative tool for assessing the similarity between simulated and experimental Chladni patterns of circular and square plates. SSIM evaluates image correspondence based on local structural relationships in pixel intensity, providing a perceptually meaningful measure of similarity that reflects geometric and spatial alignment rather than pixel-level accuracy (Wang et al., 2004). This makes it particularly suitable for vibration studies, where nodal geometry and structural symmetry are of greater significance than luminance or contrast differences (Bakurov et al., 2022; Peng et al., 2020).

The SSIM index for two images x and y is computed as (Wang et al., 2004; Bakurov et al., 2022):

$$SSIM(x, y) = \frac{(2\mu_x\mu_y + C_1)(2\sigma_{xy} + C_2)}{({\mu_x}^2 + {\mu_y}^2 + C_1)({\sigma_x}^2 + {\sigma_y}^2 + C_2)} \quad (17)$$

where $\mu_x$ and $\mu_y$ represent the mean pixel intensities of images x and y, ${\sigma_x}^2$ and ${\sigma_y}^2$ denote their variances, $\sigma_{xy}$ is the covariance, $C_1$ and $C_2$ are stabilizing constants. The SSIM score ranges from 0 to 1, where 1 indicates perfect structural similarity and 0 indicates no similarity, reflecting how closely two images align in terms of luminance, contrast and structural composition.

In this study, SSIM provides a structural comparison between:

1. numerical Bessel-function modes and experimental circular plate patterns.
2. Fourier-series modes and experimental square-plate patterns.

This quantitative approach offers a robust way to validate simulated modal shapes and assess geometry dependent vibrational behaviour.

## METHODOLOGY

### Tools and Materials

Numerical simulations and image-based analysis were implemented in Python using standard scientific libraries, including NumPy and SciPy for numerical computation, Matplotlib for visualization, and OpenCV, Pillow and scikit-image for image preprocessing and SSIM evaluation.

### Data Sources

### Numerical Simulations

Numerical Chladni patterns were generated based on the analytical solutions of the governing vibration equations for each plate geometry. For the circular plate, the Helmholtz equation (Equation (1)) was solved numerically in polar coordinates, where the radial and angular components were represented using Bessel functions as defined in Equation (8). These expressions were implemented to compute the corresponding vibration mode shapes.

For the square plate, the biharmonic plate equation (Equation (10)) was approximated using Fourier cosine series, where the numerical solution was based on the modal formulation given in Equation (16). These equations were implemented in Python to compute the spatial distribution of the vibration modes for selected resonant frequencies. The resulting numerical solutions were then visualized as Chladni patterns for comparison with experimental images.

### Experimental Data

Experimental Chladni pattern images were obtained from the Physics Demonstration Archive of the University of California, Santa Barbara (UCSB), [https://web.physics.ucsb.edu/~lecturedemonstrations/Composer/Pages/44.45.html](https://web.physics.ucsb.edu/~lecturedemonstrations/Composer/Pages/44.45.html). The images correspond to brass plates excited over a frequency range of 100-5000 Hz, with dimensions of 24 cm diameter and 0.86 mm thickness for the circular plate, and 24 cm × 24 cm with the same thickness for the square plate. These high-resolution images provide reliable experimental references for validating simulated mode shapes.

Representative experimental Chladni patterns for the circular plate are shown in Figures 1 and 2, corresponding to vibration modes (0,1) and (0,2), respectively demonstrating the radial symmetry typical of circular-plate vibration behavior (Leissa, 1969; Handayani et al., 2018).

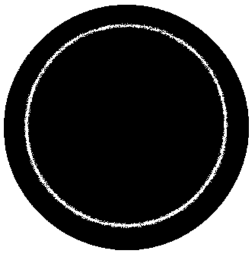

**Figure 1**. Experimental circular Chladni pattern (mode 0,1)

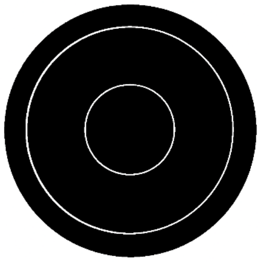

**Figure 2.** Experimental circular Chladni pattern (mode 0,2)

The square plate experimental patterns used in this study are presented in Figures 3 and 4, which illustrate modes (2,1) and (2,2). These figures provide the experimental datasets against which the numerically simulated patterns are quantitatively compared using the SSIM method.

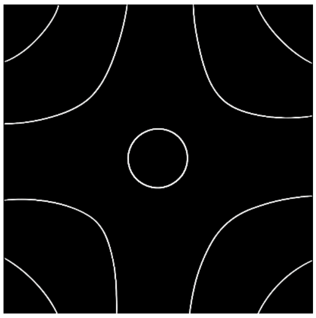

**Figure 3.** Experimental square Chladni pattern (mode 2,1)

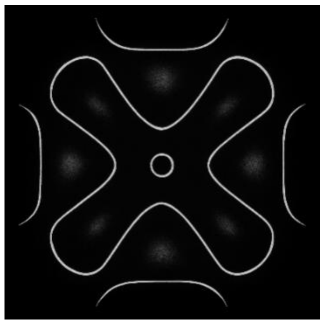

***Figure 4.*** *Experimental square Chladni pattern (mode 2,2)*

### Research Procedure

Numerical mode shapes were generated for both circular and square plates using analytical solutions derived from plate vibration theory. Corresponding experimental Chladni pattern images were then selected from the UCSB Physics Demonstration Archive. To ensure meaningful comparison between simulated and experimental patterns, all images were subjected to a uniform preprocessing pipeline using OpenCV. This preprocessing included conversion to grayscale, resizing to a standard resolution of 1500 × 1500 pixels, and contrast normalisation to reduce intensity variations across images.

The overall workflow of the image-based comparison process is illustrated in Figure 5. As shown in the figure, simulated mode shapes and experimental images were first prepared and pre-processed before being evaluated using the Structural Similarity Index Measure (SSIM). The SSIM values were computed using the *skimage.metrics.structural_similarity* function, which quantifies similarity in luminance, contrast, and structural content between image pairs (Wang et al., 2004). This procedure was repeated for each selected vibration mode to assess the correspondence between numerical predictions and experimentally observed nodal structures.

The computation of SSIM in this study follows a systematic procedure. Each image is represented as a matrix of pixel intensities after grayscale conversion. The mean pixel intensity is calculated as the average value of all pixels in the image (Wang et al., 2004):

$$\mu_x = \frac{1}{N}\sum_{i=1}^{N} x_i \qquad (18)$$

where $x_i$ is the intensity of the i-th pixel and *N* is the total number of pixels. The variance, representing the spread of pixel intensities, is computed as:

$$\sigma_x{}^2 = \frac{1}{N-1}\sum_{i=1}^{N} (x_i - \mu_x)^2 \qquad (19)$$

The covariance between two images x and y, which measures how their intensities vary together, is defined as:

$$\sigma_{xy} = \frac{1}{N-1}\sum_{i=1}^{N} (x_i - \mu_x)(y_i - \mu_y) \qquad (20)$$

where $x_i$ and $y_i$ represent the intensities of corresponding pixels in images x and y, and $\mu_x$ and $\mu_y$ are their respective mean intensities (Bakurov et al., 2022).

These statistical measures are incorporated into the SSIM formulation to evaluate similarity based on luminance, contrast and structural components. This approach ensures that the comparison between simulated and experimental Chladni patterns is quantitatively robust and directly reflects their structural correspondence.

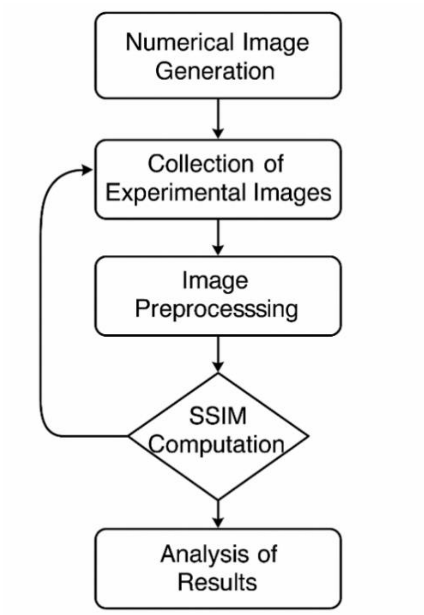


**Figure 5.** SSIM-based image comparison procedure.

## RESULTS AND DISCUSSION

Numerical simulations were performed to visualize the vibration mode shapes of circular and square plates based on analytical solutions of the governing equations. The resulting Chladni patterns form the basis for comparison with experimental images and for quantitative evaluation using the Structural Similarity Index Measure (SSIM).

For circular plates, numerical mode shapes were generated using Bessel function solutions to the Helmholtz equation (Equation (8)). The simulated patterns for modes (0,1) and (0,2), shown in Figures 6 and 7, exhibit axisymmetric standing-wave behavior. Mode (0,1) is characterized by a single nodal circle, while mode (0,2) displays two concentric nodal circles. These features are consistent with classical circular plate vibration theory and previously reported Chladni patterns.

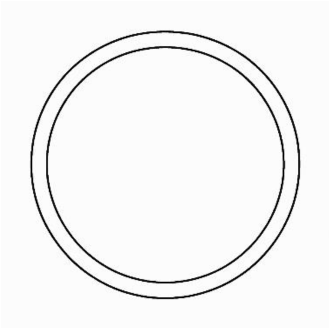

**Figure 6.** Simulated circular Chladni pattern for mode (0,1)

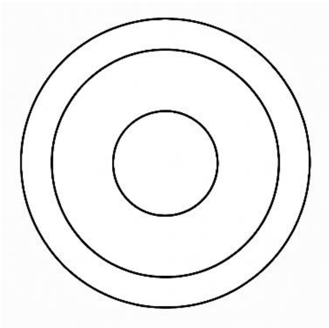

**Figure 7.** Simulated circular Chladni pattern for mode (0,2).

For square plates, numerical mode shapes were computed using cosine-based Fourier approximations to the biharmonic plate equation under idealized boundary conditions (Equation (16)). The simulated patterns for modes (2,1) and (2,2), presented in Figures 8 and 9, show nodal arrangements aligned along the Cartesian axes. Mode (2,1) exhibits two nodal divisions along one direction and one along the orthogonal direction, while mode (2,2) displays symmetric nodal divisions along both axes. These grid-like structures are consistent with analytical expectations for square plate geometries.

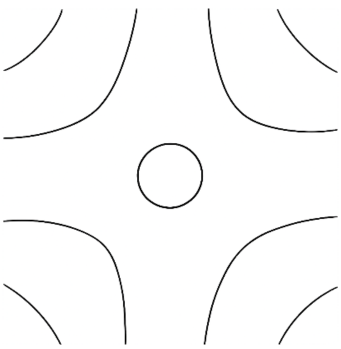

**Figure 8.** Simulated square Chladni pattern for mode (2,1)

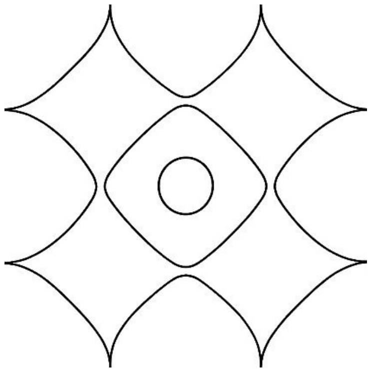

**Figure 9.** Simulated square Chladni pattern for mode (2,2)

To quantitatively assess the agreement between simulated and experimental Chladni patterns, SSIM values were computed for each corresponding vibration mode following uniform image preprocessing. The results for both matched and cross-mode comparisons are summarized in Table 1.

For circular patterns, the matched-mode SSIM values are 0.9326 for mode (0,1) and 0.9103 for mode (0,2), indicating a high level of structural similarity between numerical and experimental patterns. These values reflect the ability of the Bessel-function-based formulation to reproduce radial nodal structures with minimal deviation.

For square patterns, the matched-mode SSIM values are 0.9026 for mode (2,1) and 0.7517 for mode (2,2). While mode (2,1) shows strong agreement, the lower similarity observed for mode (2,2) corresponds to increased structural complexity. Higher-order modes introduce finer nodal features, which are more sensitive to experimental conditions such as noise, boundary irregularities, and alignment differences.

Cross-mode SSIM analysis further illustrates the behavior of the similarity metric. Circular plates exhibit low cross-mode similarity (0.3906), indicating clear structural distinction between modes. In contrast, square plates show higher cross-mode similarity (0.8311), which reflects partial structural overlap arising from shared Cartesian symmetry and similar global nodal alignment.

These observations indicate that SSIM effectively captures structural differences between vibration modes while remaining sensitive to geometric symmetry. The metric distinguishes well between distinct radial modes in circular plates, while for square plates, global symmetry contributes to higher similarity even when local nodal structures differ.

**Table 1.** SSIM values for matched and unmatched Chladni mode comparisons

| Comparison type | Plate | Mode | SSIM score |
|---|---|---|---|
| Matched Modes | Circular | (0,1) | 0.9326 |
| | | (0,2) | 0.9103 |
| | Square | (2,1) | 0.9026 |
| | | (2,2) | 0.7517 |
| Unmatched Modes | Circular | (0,1) vs (0,2) | 0.3906 |
| | Square | (2,1) vs (2,2) | 0.8311 |

The numerical results obtained in this study are consistent with previous research on plate vibration in which the governing equations are solved analytically and numerically. For circular plates, the solution of the Helmholtz equation (Equation (8)) using Bessel functions produces nodal patterns that agree with those reported by Handayani et al. (2018), where similar numerical approaches were used to simulate resonant modes. The high SSIM values obtained for modes (0,1) and (0,2) quantitatively confirm that the numerical solution accurately reproduces the radial nodal structures observed in experimental data.

For square plates, the Fourier-based solution of the biharmonic plate equation (Equation (16)) generates nodal patterns consistent with the results reported by D'Alessio (2021), where the equation is solved under comparable boundary conditions. The reduced SSIM value observed for mode (2,2) reflects the increased sensitivity of higher-order numerical solutions to boundary conditions and experimental imperfections, which affect the spatial distribution captured in the image-based comparison.

The contribution of this study lies in extending the validation of numerically solved vibration equations through a quantitative image-based approach. By applying SSIM to compare simulated and experimental patterns, this work provides a direct link between numerical solutions of governing equations and observable physical structures. This approach can be applied in other areas of physics involving wave phenomena, such as acoustics, optics, and structural dynamics, where numerical solutions require validation against experimental or visual data.

## CONCLUSION

This study demonstrates that the Structural Similarity Index Measure (SSIM) provides a reliable and practical metric for quantitatively evaluating the correspondence between simulated and experimental Chladni patterns for circular and square plates. Numerical mode shapes derived from Bessel-function and cosine-series solutions show strong agreement with experimental observations, as indicated by high similarity values for matched modes. In contrast, lower similarity scores for unmatched modes confirm the ability of SSIM to distinguish between different modal configurations, while the relatively higher cross-mode similarity observed in square plates reflects its sensitivity to shared geometric symmetry.

These findings establish SSIM as an effective framework for validating plate vibration models and for linking computational simulations with experimental visualization. Future work may extend this approach by integrating SSIM with complementary image analysis techniques to enhance mode discrimination and improve robustness in the analysis of higher-order or more complex vibration patterns.

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